Mechanical Vibrations

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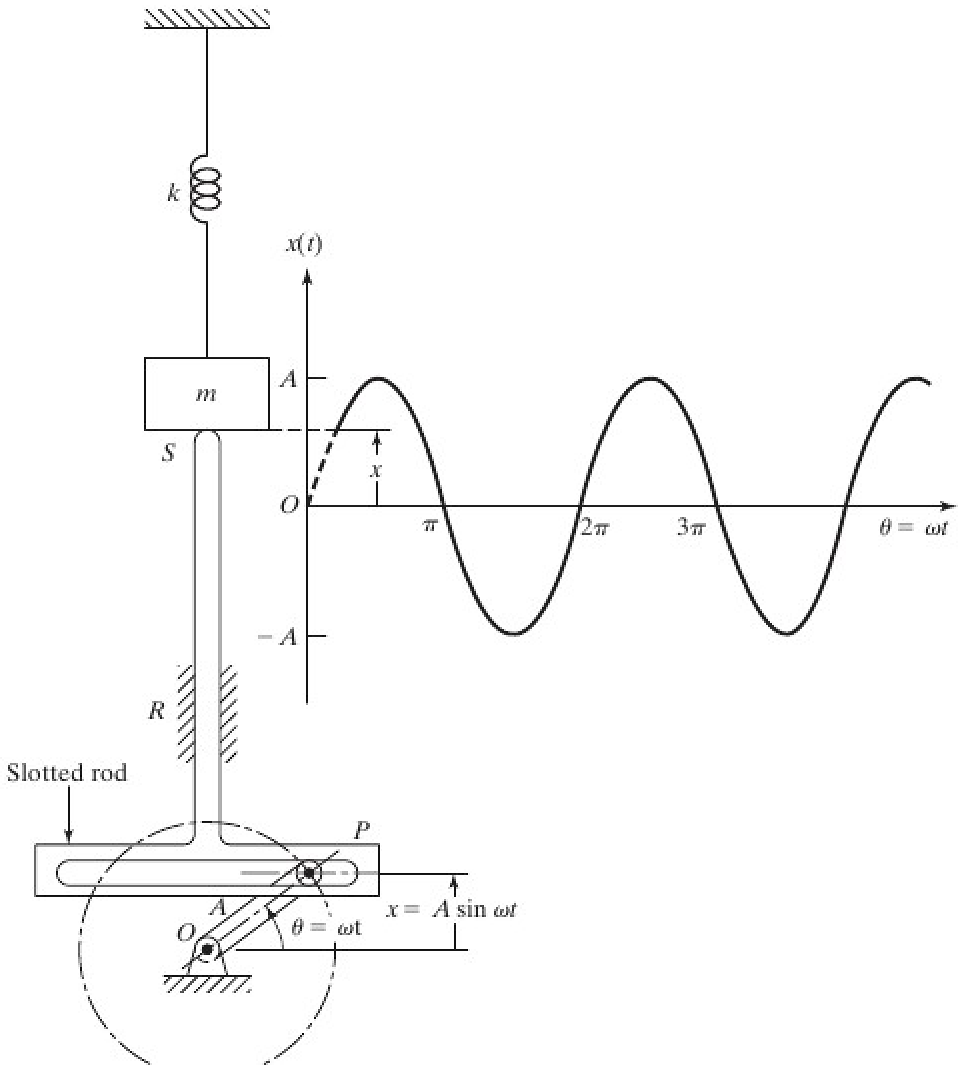
Harmonic Motion

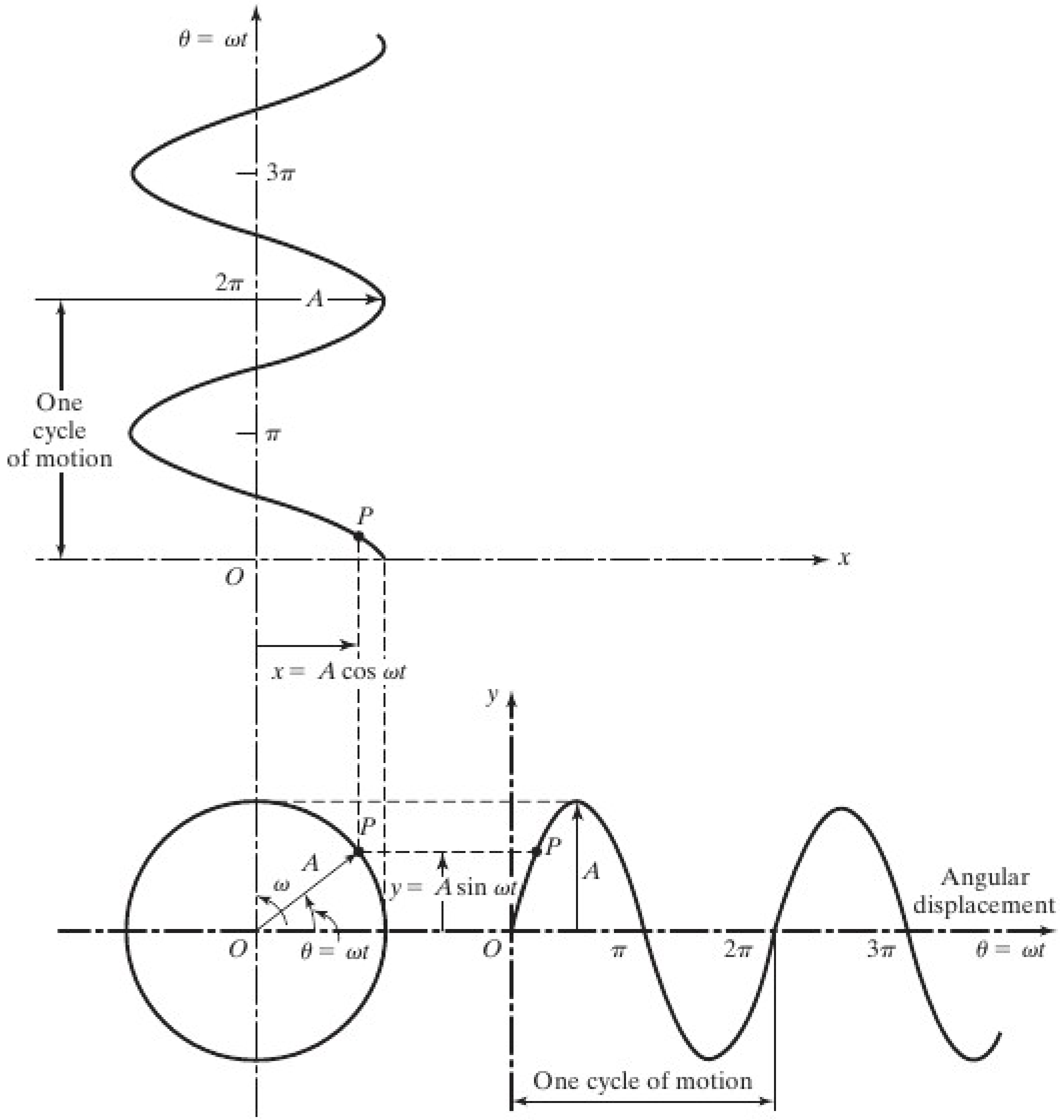


$$x(t) = A\sin(\theta) = A\sin(\omega t)$$

$$\frac{dx(t)}{dt} = \omega A\cos(\omega t)$$

$$\frac{d^2x(t)}{dt^2} = -\omega^2 A\sin(\omega t)$$

Harmonic Motion



$$y = A\sin(\omega t)$$

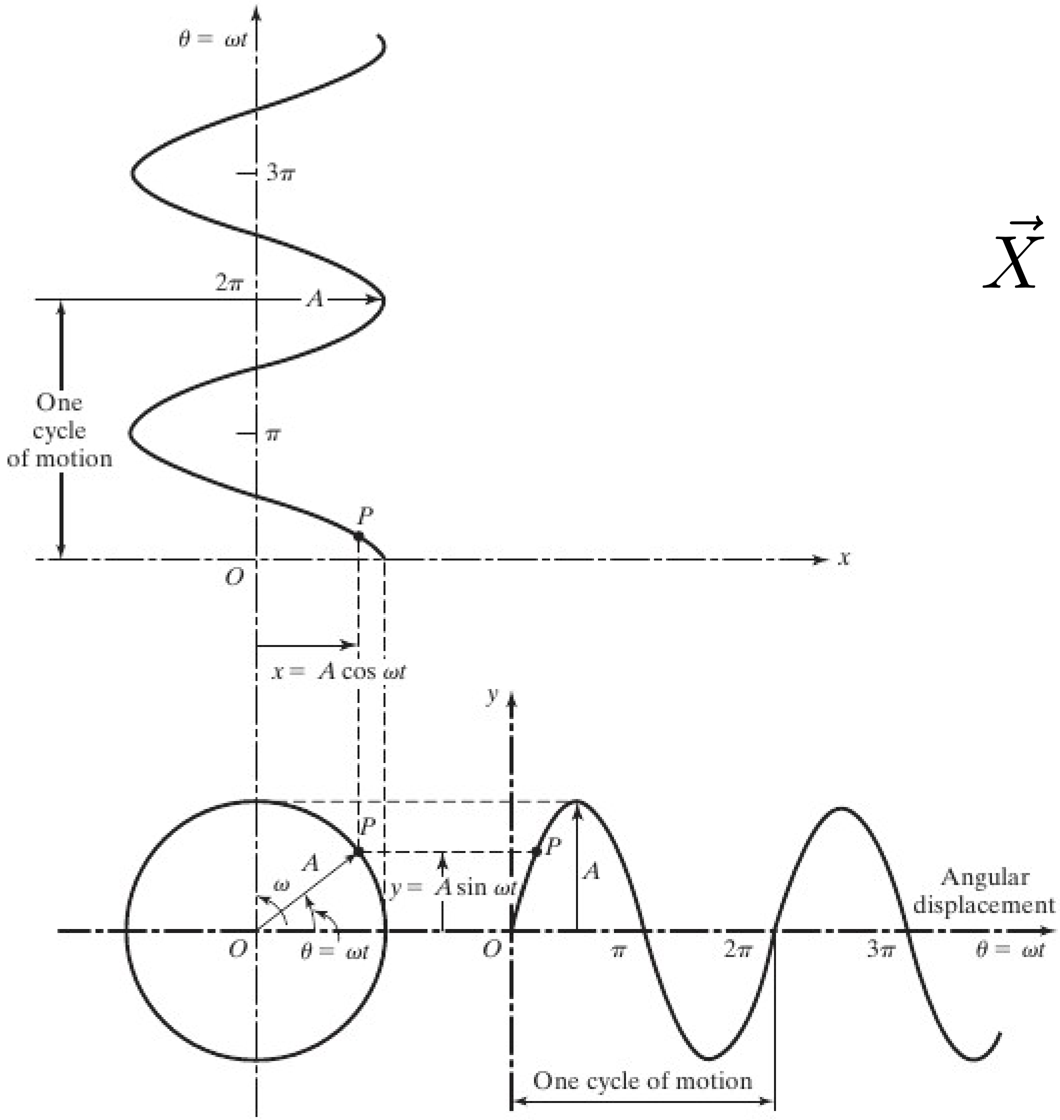
$$x = A\cos(\omega t)$$

$$\vec{X} = \vec{OP}$$

$$\vec{X} = a + ib$$

$$\vec{X} = A\cos(\omega t) + iA\sin(\omega t)$$

Harmonic Motion



$$\vec{X} = a + ib$$

$$\vec{X} = A\cos(\omega t) + iA\sin(\omega t)$$

$$A = \sqrt{a^2 + b^2}$$

$$\theta = \arctan\left(\frac{b}{a}\right)$$

Harmonic Motion

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots = 1 + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^4}{4!} + \cdots$$

$$i\sin\theta = i\left[\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right] = i\theta + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^5}{5!} + \cdots$$

$$(\cos\theta + i\sin\theta) = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \cdots = e^{i\theta}$$

$$(\cos\theta - i\sin\theta) = 1 - i\theta + \frac{(i\theta)^2}{2!} - \frac{(i\theta)^3}{3!} + \cdots = e^{-i\theta}$$

Addition of Harmonics (ex. 1.18)

Find de sum of two harmonics

$$x_1(t) = 10\cos(\omega t) \qquad x_2(t) = 15\cos(\omega t + 2)$$

$$x(t) = x_1(t) + x_2(t) = A\cos(\omega t + \alpha)$$

Addition of Harmonics (ex. 1.18)

$$A\cos(\omega t+\alpha) = x_1(t) + x_2(t)$$

$$A\cos(\omega t+\alpha) = 10\cos(\omega t) + 15\cos(\omega t+2)$$

$$A\left[\cos(\omega t)\cos(\alpha) - \sin(\omega t)\sin(\alpha)\right] = 10\cos(\omega t) + 15\left[\cos(\omega t)\cos(2) - \sin(\omega t)\sin(2)\right]$$

$$A\cos(\alpha) = 10 + 15\cos(2)$$
$$A\sin(\alpha) = 15\sin(2)$$

Addition of Harmonics (ex. 1.18)

$$A\cos(\alpha) = 10 + 15\cos(2)$$
$$A\sin(\alpha) = 15\sin(2)$$

$$A = \sqrt{(10 + 15\cos(2))^2 + (15\sin(2))^2} = 14.1477$$

$$\alpha = \arctan\left(\frac{15\sin(2)}{10 + 15\cos(2)}\right) = 74.5963^o$$

Problems

1. Express the number $10 + 20i$ in terms of $Ae^{i\theta}$

2. Express the sum of $x_1 = 1 + 2i$, $x_2 = 2 - 6i$ in terms of $Ae^{i\theta}$

3. Find the some of two harmonic motions

$$x_1 = 5\cos(3t + 1) \qquad x_2 = 10\cos(3t + 2)$$

The phenomenon of Beats

Beats. When two harmonic motions, with frequencies close to one another, are added, the resulting motion exhibits a phenomenon known as beats. For example, if

$$x_1 = X\cos(\omega t)$$
$$x_2 = X\cos((\omega+\delta)t)$$

where δ is a small quantity, the addition of these motions yields

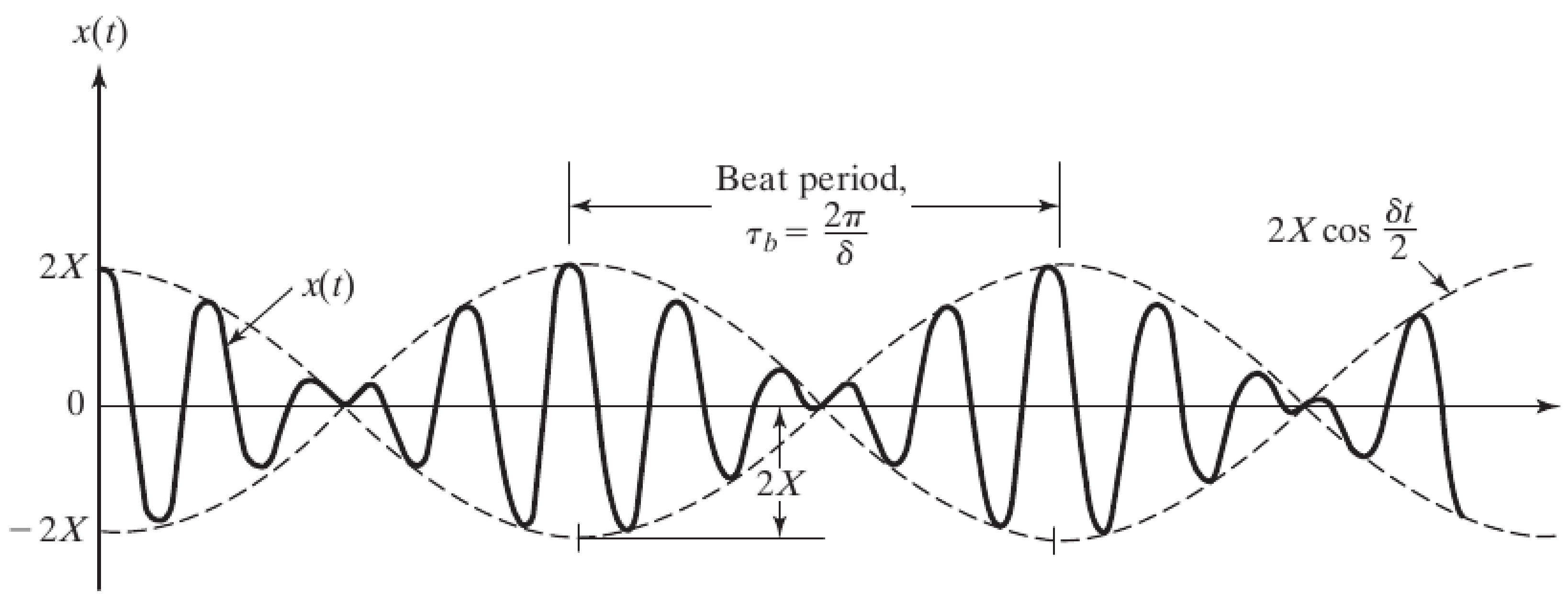
$$x(t) = x_1(t) + x_2(t) = X[\cos\omega t + \cos(\omega + \delta)t]$$

The phenomenon of Beats

Using the relation

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

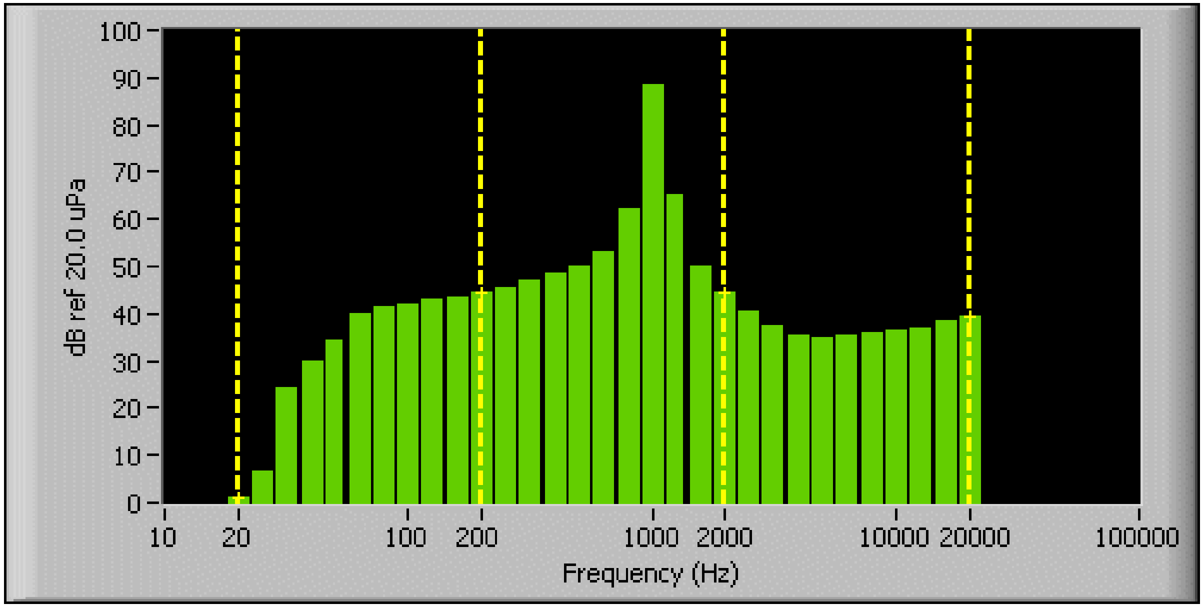
$$x(t) = 2X\cos\frac{\delta t}{2}\cos\left(\omega + \frac{\delta}{2}\right)t$$



Concepts

Octave. When the maximum value of a range of frequency is twice its minimum value, it is known as an octave band. For example, each of the ranges 75-150 Hz, 150-300 Hz, and 300-600 Hz can be called an octave band. In each case, the maximum and minimum values of frequency, which have a ratio of 2:1, are said to differ by an octave.

Concepts



100
90
80
70
60
50
40
30
20
10
0
dB ref 20.0 uPa
10
20
100
200
1000
2000
10000
20000
100000
Frequency (Hz)

Concepts

Decibel. The various quantities encountered in the field of vibration and sound (such as displacement, velocity, acceleration, pressure, and power) are often represented using the notation of decibel. A decibel (dB) is originally defined as a ratio of electric powers:

$$\mathrm{dB} = 10 \log\left(\frac{P}{P_0}\right)$$

Concepts

where P_0 is some reference value of power. Since electric power is proportional to the square of the voltage (X), the decibel can also be expressed as

$$\text{dB} = 10 \log\left(\frac{X}{X_0}\right)^2 = 20 \log\left(\frac{X}{X_0}\right)$$

where X_0 is a specified reference voltage. In practice, this equation is also used for expressing the ratios of other quantities such as displacements, velocities, accelerations, and pressures.

Harmonic Analysis

Fourier Series Expansion

$$x(t) = \frac{a_0}{2} + a_1 \cos \omega t + a_2 \cos 2\,\omega t + \cdots$$
$$+ b_1 \sin \omega t + b_2 \sin 2\,\omega t + \cdots$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

Harmonic Analysis

Fourier Series Expansion

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_0 = \frac{\omega}{\pi}\int_0^{2\pi/\omega} x(t)\ dt = \frac{2}{\tau}\int_0^{\tau} x(t)\ dt$$

$$a_n = \frac{\omega}{\pi}\int_0^{2\pi/\omega} x(t)\ \cos n\omega t\, dt = \frac{2}{\tau}\int_0^{\tau} x(t)\ \cos n\omega t\, dt$$

$$b_n = \frac{\omega}{\pi}\int_0^{2\pi/\omega} x(t)\ \sin n\omega t\, dt = \frac{2}{\tau}\int_0^{\tau} x(t)\ \sin n\omega t\, dt$$

Harmonic Analysis

Fourier Series Expansion