Mechanical Vibrations

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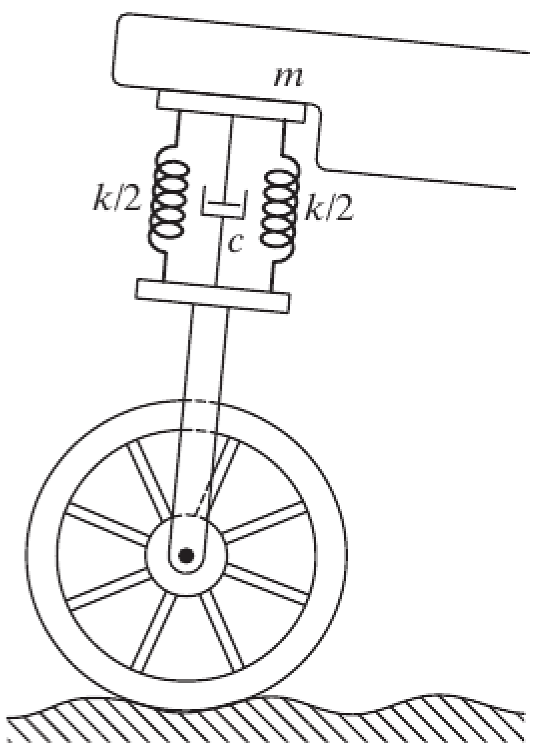
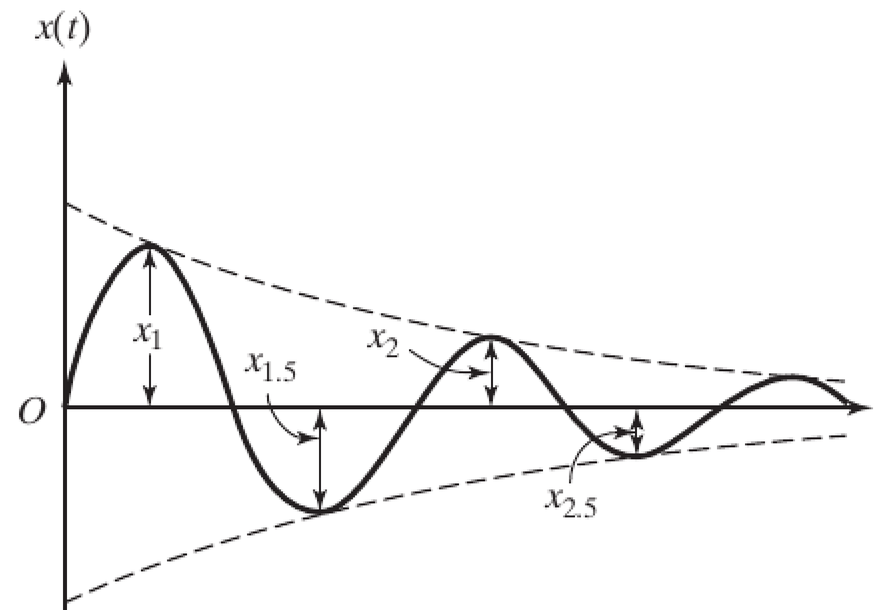
Shock Absorber for a Motorcycle

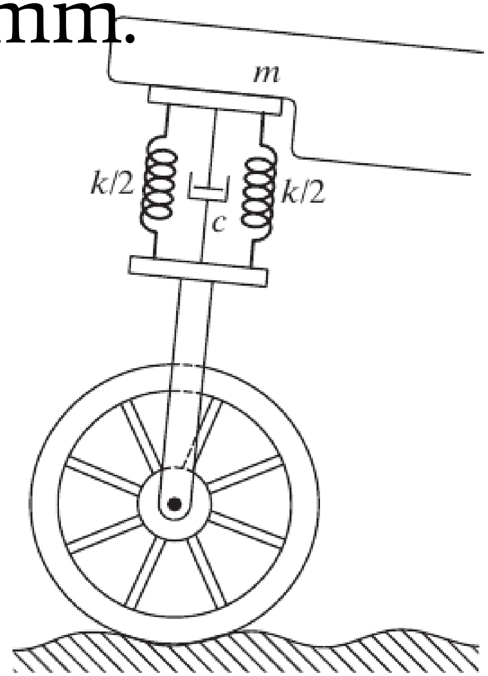
An underdamped shock absorber is to be designed for a motorcycle of mass 200 kg. When the shock absorber is subjected to an initial vertical velocity due to a road bump, the resulting displacement-time curve is shown in the figure.

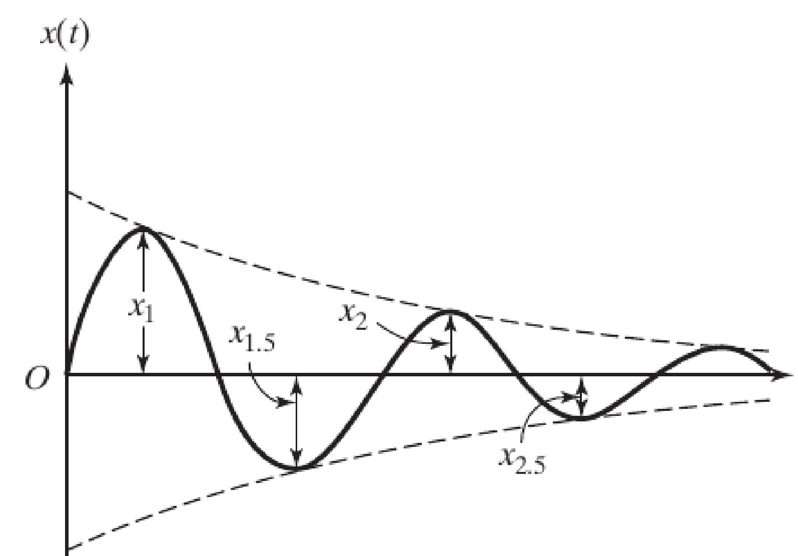




Shock Absorber for a Motorcycle

Find the necessary stiffness and damping constants of the shock absorber if the damped period of vibration is to be 2s and the amplitude x1 is to be reduced to one-fourth in one half cycle (i.e., $x_{1.5} = x_{1/4}$). Also find the minimum initial velocity that leads to a maximum displacement of 250 mm.





Shock Absorber for a Motorcycle

Solution:

Since $x_{1.5} = x_1/4$, $x_2 = x_{1.5}/4 = x_1/16$

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln(16) = 2.7726$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \longrightarrow \zeta = 0.4037$$

Shock Absorber for a Motorcycle

Solution:

$$\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} = 2$$

$$\omega_n = \frac{2\pi}{2\sqrt{1-(0.4037)^2}} = 3.4338 \text{ rad/s}$$

Shock Absorber for a Motorcycle

Critical Damping:

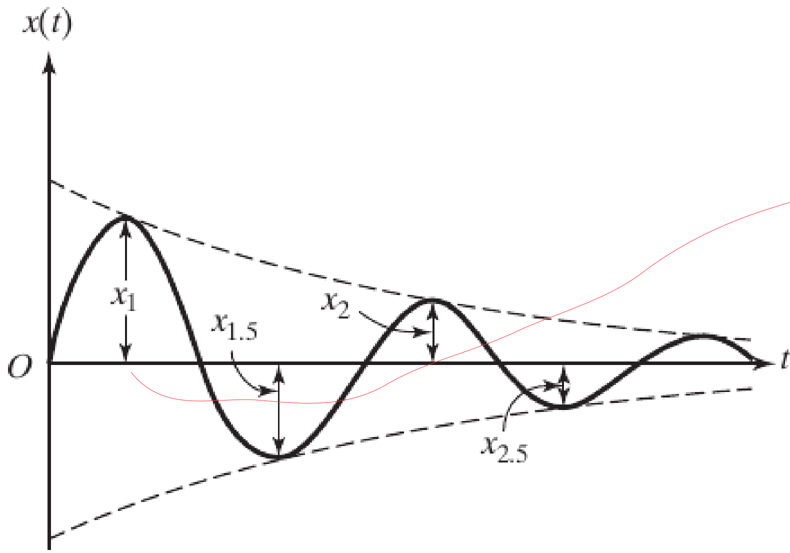
$$c_c = 2m\omega_n = 2(200)(3.4338) = 1373.54 \text{ N-s/m}$$

Damping Constant:

$$c = \zeta c_c = (0.4037)(1373.54) = 554.4981 \text{ N-s/m}$$

Shock Absorber for a Motorcycle

The displacement is a maximum at → t_1



$$\sin \omega_d t_1 = \sqrt{1 - \zeta^2}$$

$$\sin \omega_d t_1 = \sin \pi t_1 = \sqrt{1 - (0.4037)^2} = 0.9149$$

$$t_1 = \frac{\sin^{-1}(0.9149)}{\pi} = 0.3678 \text{ sec}$$

Shock Absorber for a Motorcycle

The envelope passing through the maximum points

$$x = \sqrt{1 - \zeta^2} X e^{-\zeta \omega_n t}$$

Since

$$x = 250 \text{ mm}$$

$$0.25 = \sqrt{1 - (0.4037)^2}\, X e^{-(0.4037)(3.4338)(0.3678)}$$

$$X = 0.4550 \text{ m}$$

Shock Absorber for a Motorcycle

The velocity of the mass can be obtained

$$x(t) = Xe^{-\zeta\omega_n t}\sin\omega_d t$$

$$\dot{x}(t) = Xe^{-\zeta\omega_n t}(-\zeta\omega_n \sin\omega_d t + \omega_d \cos\omega_d t)$$

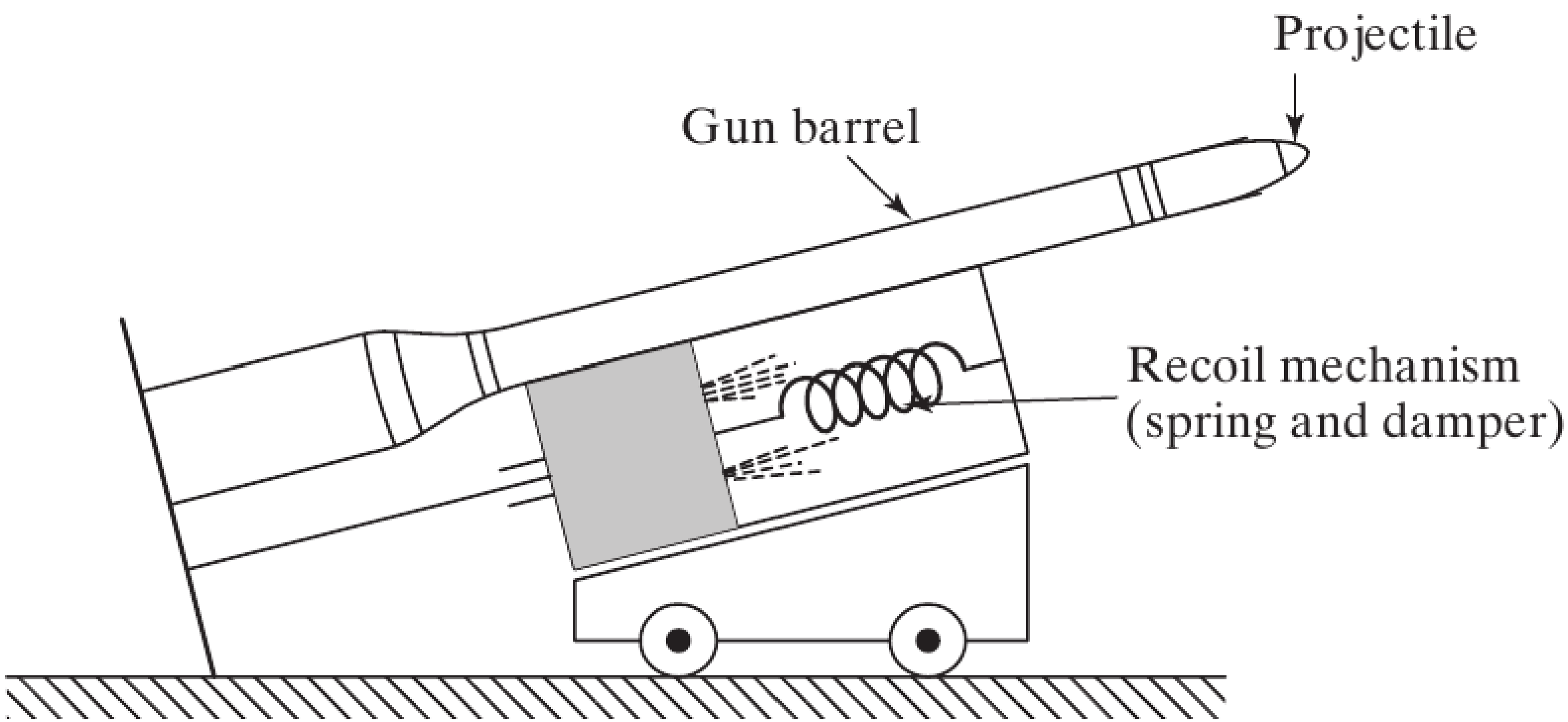
$$\dot{x}(t=0) = \dot{x}_0 = X\omega_d = X\omega_n\sqrt{1-\zeta^2}$$

$$= (0.4550)(3.4338)\sqrt{1-(0.4037)^2}$$

$$= 1.4294 \text{ m/s}$$

Analysis of Cannon

When the gun is fired, high pressure gases accelerate the projectile inside the barrel to a very high velocity. The reaction force pushes the gun barrel in the direction opposite that of the projectile.



Analysis of Cannon

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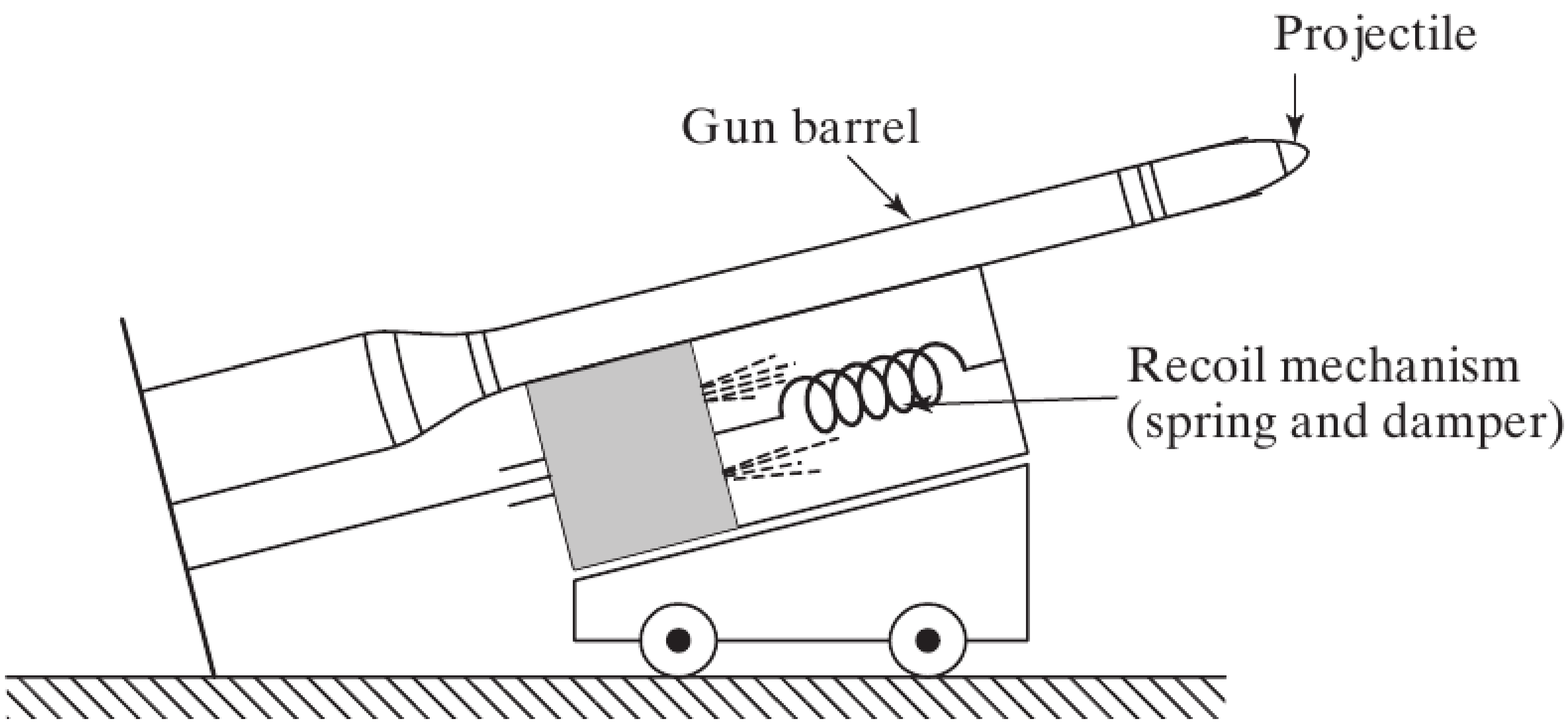
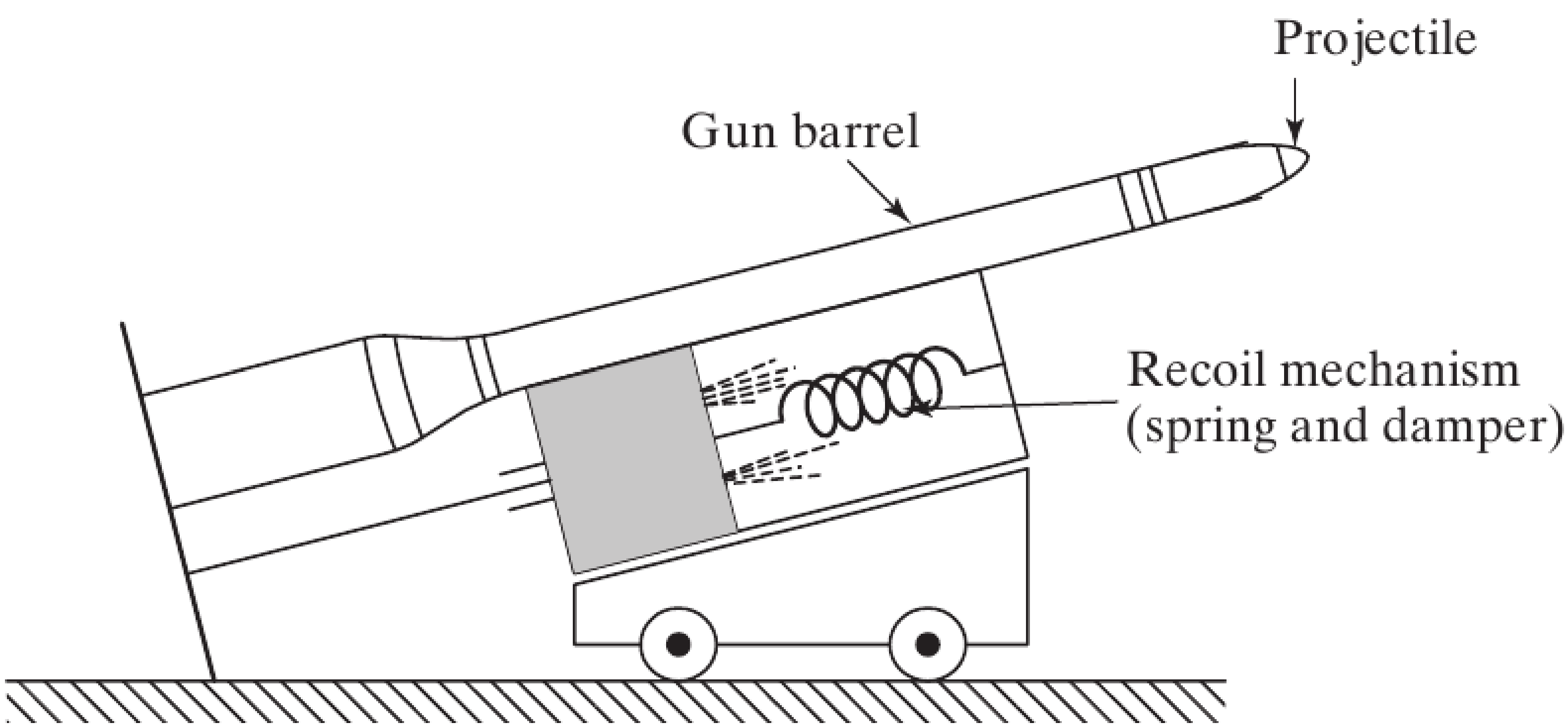


Analysis of Cannon

Since it is desirable to bring the gun barrel to rest in the shortest time without oscillation, it is made to translate backward against a critically damped spring-damper system called the recoil mechanism.



Analysis of Cannon

In a particular case, the gun barrel and the recoil mechanism have a mass of 500 kg with a recoil spring of stiffness 10,000 N/m. The gun recoils 0.4 m upon firing.

Find

(1) the critical damping coefficient of the damper
(2) the initial recoil velocity of the gun
(3) the time taken by the gun to return to a position 0.1 m from its initial position.

Analysis of Cannon

The undamped natural frequency

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10{,}000}{500}} = 4.4721 \text{ rad/s}$$

The critical damping coefficient

$$c_c = 2m\omega_n = 2(500)(4.4721) = \boxed{4472.1 \text{ N-s/m}}$$

Analysis of Cannon

The response of critically damped system

$$x(t) = (C_1 + C_2 t)e^{-\omega_n t}$$

$$C_1 = x_0 \qquad\qquad C_2 = \dot{x}_0 + \omega_n x_0$$

To find the time when the displacement is a maximum

$$\dot{x}(t) = 0$$

$$\dot{x}(t) = C_2 e^{-\omega_n t} - \omega_n (C_1 + C_2 t) e^{-\omega_n t}$$

Analysis of Cannon

$$\dot{x}(t) = C_2 e^{-\omega_n t} - \omega_n (C_1 + C_2 t) e^{-\omega_n t} = 0$$

$$t_1 = \left(\frac{1}{\omega_n} - \frac{C_1}{C_2} \right)$$

$$x_0 = C_1 = 0$$

$$t_1 = 1/\omega_n$$

Analysis of Cannon

$$x_{\max} = 0.4 \text{ m}$$

$$x_{\max} = x(t = t_1) = C_2 t_1 e^{-\omega_n t_1} = \frac{\dot{x}_0}{\omega_n} e^{-1} = \frac{\dot{x}_0}{e\omega_n}$$

$$\dot{x}_0 = x_{\max}\omega_n e = (0.4)(4.4721)(2.7183)$$

$$= \boxed{4.8626 \text{ m/s}}$$

→ Initial Velocity

Analysis of Cannon

the time taken by the gun to return to a position 0.1 m from its initial position.

$$x(t) = (C_1 + C_2 t) e^{-\omega_n t}$$

$$0.1 = C_2 t_2 e^{-\omega_n t_2} = 4.8626 t_2 e^{-4.4721 t_2}$$

Use a numerical root find Method to obtain the solution

$$t_2 = 0.8258 \text{ s.}$$

Stability of Mechanical Systems

Considering the spring-mass-damper system

$$m\ddot{x} + c\dot{x} + kx = 0$$

whose characteristic equation can be expressed as

$$ms^2 + cs + k = 0$$

or

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Stability of Mechanical Systems

$$ms^2 + cs + k = 0$$

$$s_1, s_2 = \frac{-c \pm \sqrt{c^2 - 4mk}}{2\,m}$$

or

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_1, s_2 = -\zeta\omega_n \pm i\omega_n\sqrt{1 - \zeta^2}$$

Stability of Mechanical Systems

The solution can be written as

$$x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

Stability of Mechanical Systems