
Special Report

Spandrel Beam Behavior and Design



Charles H. Rath

Senior Principal
Raths, Raths & Johnson, Inc.
Structural Engineers
Willowbrook, Illinois

Presents common precast spandrel beam distress causes, discusses types of loads applied to spandrel beams and overall torsion equilibrium requirements, provides design relationships for spandrel beams, offers design criteria for spandrel beam connections, sets forth basic good design practices for spandrel beams, and gives design examples.

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Spandrel beams, reinforced or prestressed, are important and functional elements of precast concrete structures. Given the current design knowledge presented by the PCI Design Handbook¹ and the PCI Connections Manual,² and basic fundamentals of structural engineering mechanics, it would seem the topic of spandrel beam design does not merit further review. Yet, considering the number of precast framed structures experiencing various types of problems and distress with spandrel beams, something is amiss regarding the designer's understanding of spandrel beam behavior and design requirements.

The purpose of this paper is to review some of the common problems typically associated with spandrel beams, and to suggest design requirements for them. Towards this end, discussions herein are devoted to actual problems experienced by spandrel beams, load supporting functions, general design requirements, connections, and good design practice. While spandrel beams are used to satisfy a variety of structural functions, this presentation will emphasize simple span load supporting members.

PAST AND CURRENT PROBLEMS

Difficulties related to spandrel beams occur for members supporting floor loads, those which are part of a moment frame, or members that are neither gravity supporting nor part of a moment resisting frame. Typically, simple span spandrel beams as utilized in parking garages appear to be the most problem prone although similar troubles develop in office or other "building" type structures.

The difficulties and problems discussed usually do not result in the collapse of spandrel beam members. Nevertheless, these problems can create

substantial repair costs, construction delays, temporary loss of facility use, and various legal entanglements.

Types of Problems

The types of problems experienced by spandrel beams can be categorized as follows:

- Overall torsional equilibrium of the spandrel beam as a whole.
- Internal torsion resulting from beams not being loaded directly through their shear center.
- Member end connection.
- Capability of the spandrel beam's ledge and web to support vertical loads.
- Volume change restraint forces induced into spandrel beams.

Actual Problems

The following discussion of actual case histories relates to projects where photographs can be used to illustrate the problems associated with the above listed items. This is not meant to imply that aspects not covered by the case histories are less significant than those considered.

The conditions associated with lack of overall beam torsion equilibrium are shown in Fig. 1. The non-alignment of applied loads and beam end reactions can be seen in Fig. 1a. Fig. 1b shows the beam rotation crushing of the topping concrete caused by the lack of beam and torsional equilibrium connections. In Fig. 1, the torsional rotation problem is compounded by the presence of neoprene type bearing pads at the beam and tee bearings where the pad deformation causes further torsional roll. Generally, the problem of overall spandrel beam torsional equilibrium represents the majority of the difficulties experienced by spandrel beams.

Internal spandrel beam torsion distress, frequently within distance " d " of the spandrel beam's ends, is often observed.* Fig. 2a illustrates a spandrel



Fig. 1a. Lack of overall torsion equilibrium: underside view showing non-alignment of applied loads and end reaction.

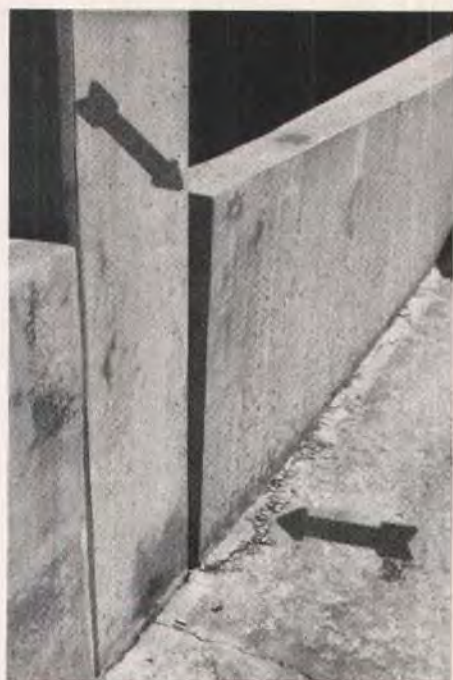


Fig. 1b. Lack of overall torsion equilibrium: top view showing beam rotation and crushing of topping (arrows).

beam loaded eccentric to its shear center and shows the beam's torsional equilibrium connections (note the tee legs on either side of the column result in a concentrated load within distance " d "). Internal torsion cracks in the spandrel beam web can be observed in Fig. 2b. These torsion cracks result from a combination of internal beam torsion and the flexural behavior influence of the beam end torsion equilibrium connections. Torsion distress (cracking) at beam ends is a common problem affecting spandrel beams when designs do not consider the influence of loads within distance " d " or torsion equilibrium connection forces.

*Note: " d " denotes the distance from the extreme compression fiber of the member to the centroid of flexural tension reinforcement.

The lack of overall spandrel beam torsion equilibrium is often reflected at column support connections resulting in high nonuniform bearing stresses caused by spandrel beam torsion roll. Fig. 3a shows the applied tee loads not aligning with the support column reaction. Lack of required overall beam torsion equilibrium connections during erection often results in excessively high (usually localized) bearing distress as reflected by Fig. 3b. A variation of the same problems caused by spandrel beam torsion roll (lack of beam end torsion equilibrium connections) is presented by Fig. 4a which produced column corbel failures. Fig. 4b indicates the magnitudes of torsion roll that can develop bearing distress when the necessary torsion equilibrium connections are not provided.



Fig. 2a. Underside view of beam end torsion cracking within distance "d" caused by equilibrium connections and internal torsion (arrow indicates location of reaction).



Fig. 2b. Top view of end cracks (arrows).

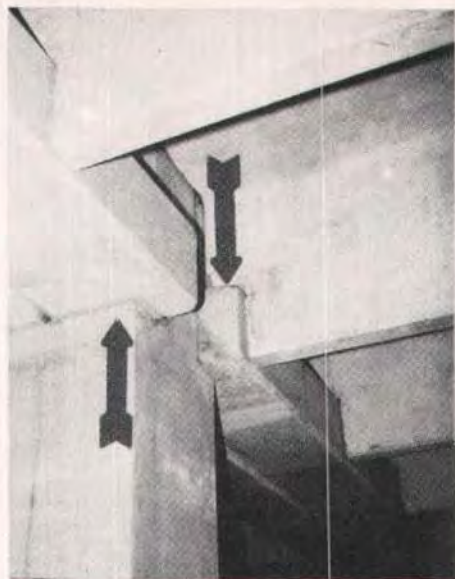


Fig. 3a. Underside view of bearing distress resulting from torsional roll due to lack of erection connections (left arrow points to beam reaction and right arrow indicates applied loads).



Fig. 3b. Close-up of typical bearing distress.



Fig. 4a (left). Upward view of column corbel failure caused by lack of any overall torsion equilibrium connections.

Fig. 4b (above). Example of torsional roll magnitude at a non-distressed corbel.



Fig. 5. Beam ledge end corbel failure (applied loads and reaction in alignment).

An example of distress resulting from inadequate considerations of structural behavior, even though overall torsion equilibrium is assured by applied loads and beam reactions aligning, is given by Fig. 5. The spandrel beam haunch, at the beam end, acts like a corbel, and when not reinforced to resist the entire beam reaction, as in Fig. 5, distress or failure develops.



Fig. 6a. Overall view of torsion equilibrium tension insert connection (top arrow indicates bearing pad location and lower arrow points to failed tension insert).



Fig. 6b. Close-up view of failure (left arrow points to insert and right arrow indicates shear cone crack failure plane).

Figs. 6a and 6b show yet another variation of spandrel distress resulting from a failure of the overall torsion equilibrium tension insert connection. The non-alignment of the beam end reaction and the applied tee loads causing torsion were resisted by a horizontal couple developed by a steel bolted insert tension connection near the top of the ledge and bearing of the beam web against the column top.

The dap behavior of spandrel beam haunches (or ledges), particularly at beam ends, sometimes is neglected resulting in haunch failures as shown in Fig. 7. This particular member (Fig. 7) failed because the necessary reinforcement was not present at the beam end, and because the concrete acting as plain concrete did not have the necessary capacity to resist applied shear and flexure forces. Another category of spandrel dap and internal torsion problems is shown by Fig. 8, where insufficient dap reinforcement, A_{sh} , was used (refer to Fig. 40 and the Notation for the full meaning of A_{sh}).

Spandrel beams employing relatively thin webs have, depending on the web thickness and the type of web reinforcement (shear and torsion), suffered separation of the entire ledge from the beam as illustrated by Fig. 9. This type of distress results from inadequate considerations of how tee reaction loads to the ledge are transmitted into the beam web. The ledge transfer mechanism resembles a dap (an upside down dap) subject to both flexure and direct tension.

The actual case histories reviewed, and many others not commented upon, indicate that some designers do not understand the entire behavior of spandrel beams. The problems related to spandrel behavior occur in all types of structures, are not confined to any one geographic area, and inevitably result when one or more of the basic engineering fundamentals are missed or ignored.



Fig. 7. Ledge failure near beam end.



Fig. 8. End support dap distress caused by insufficient dap and torsion reinforcement (upper arrow indicates epoxied dap cracks and lower arrow points to modified end support).



Fig. 9. Complete separation failure of ledge from beam web.

TYPES OF APPLIED LOADS

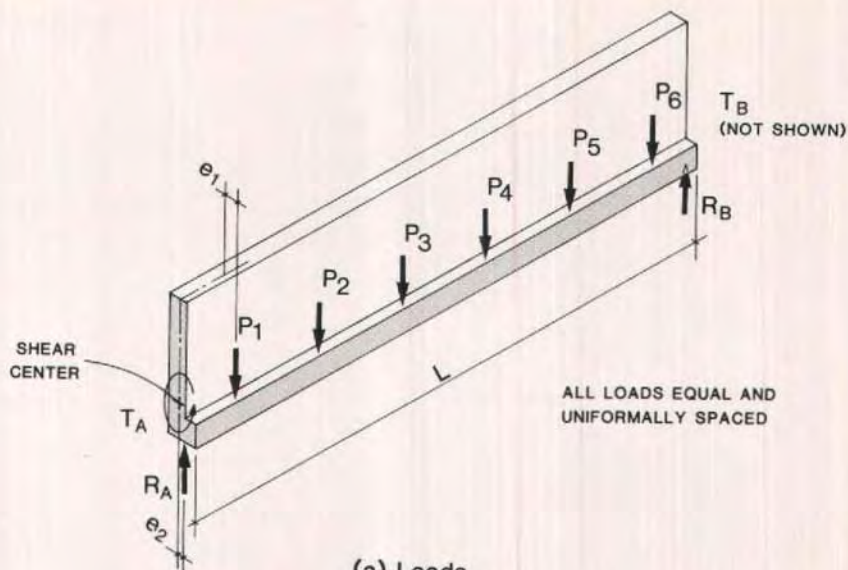
Spandrel beams are subjected to a variety of loads. These loads result from applied gravity forces, horizontal impact forces (e.g., parking garage spandrels), end connections, ledges transmitting loads to the spandrel beam, volume change forces, and frame moments. These loading cases can act separately or in combination. The discussions herein relate to precast spandrel beams acting as simple span load supporting members.

Gravity Loads — General Beam Loading

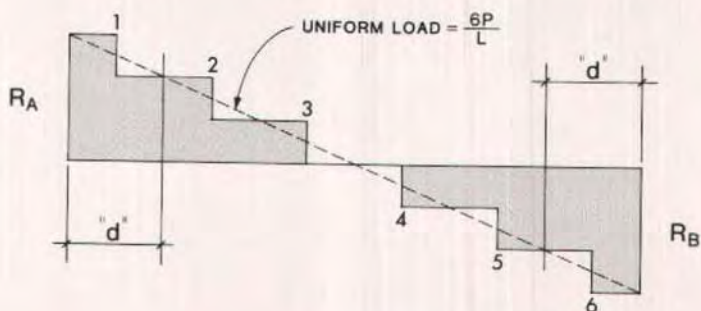
The gravity loads of a precast spandrel beam are typically concentrated and result principally from its tee legs as shown in Fig. 10. Fig. 10a presents the overall equilibrium requirements for concentrated loads applied to a simple span spandrel beam. Figs. 10b and 10c show the resulting beam shear and

internal torsion, respectively, caused by gravity loads eccentric to the beam's shear center.

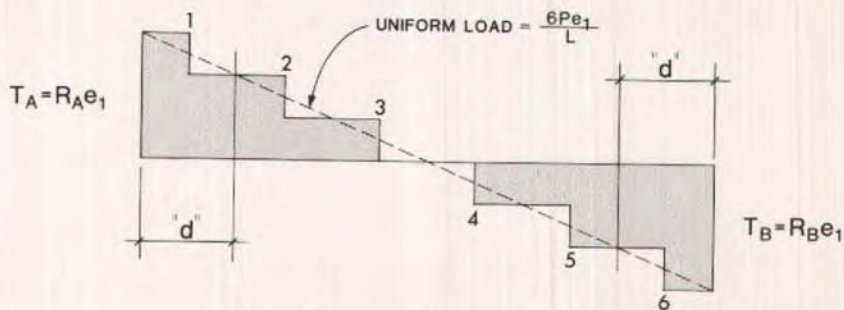
Fig. 10 shows the elements which are fundamental to the load behavior response of spandrel beams. Relative to Fig. 10a, if e_e equals or exceeds e_1 (end reactions align with applied loads when $e_1 = e_2$), then no connections are required to develop the resisting torque T_A necessary for overall gravity torsion equilibrium. The influence of loads within distance "d" for spandrel beams of significant height is shown in Figs. 10b and 10c. ACI 318-77³ Sections 11.1.3.1 and 11.1.3.2 indicate that the design for shear need only consider shears at "d" or "d/2" while Section 11.6.4 indicates that torsion, for non-prestressed members, need only be considered at distance "d" or beyond. The ACI 318-77 Code, relative to spandrel beam design, is incorrect (though not the Commentary) since using the Code procedures can result in precast span-



(a) Loads

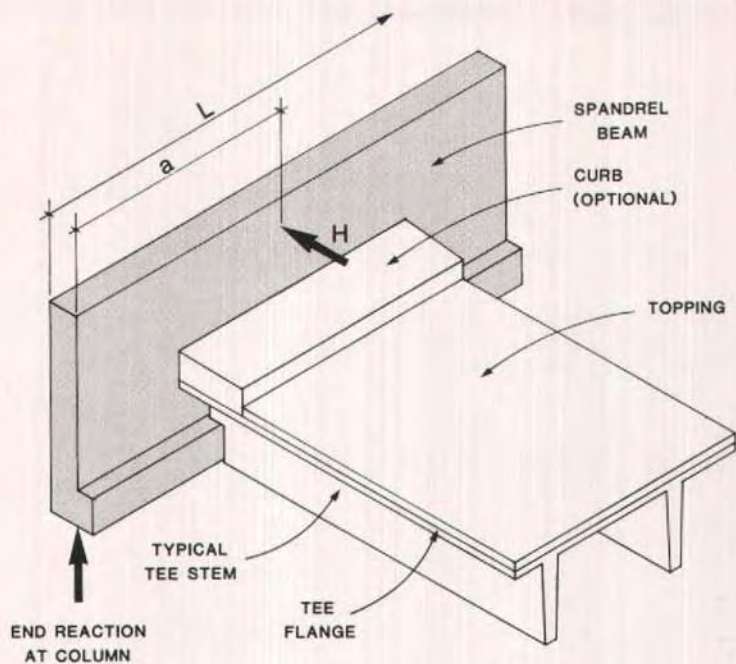


(b) Applied Shear

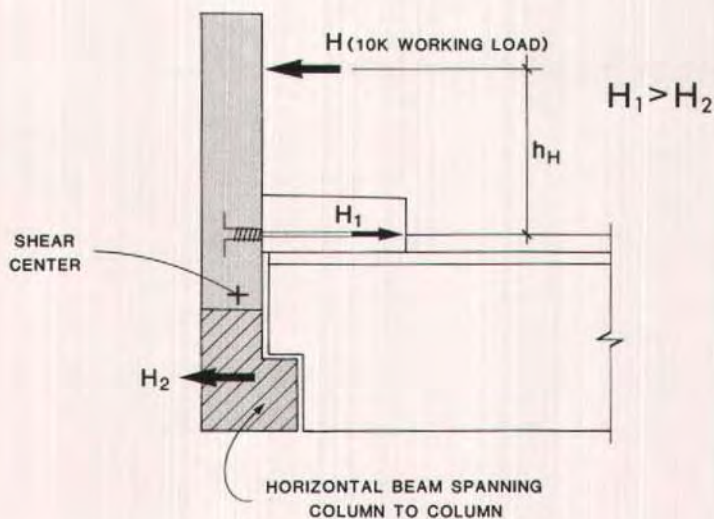


(c) Applied Torsion

Fig. 10. General gravity loads showing applied shear and torsion diagrams.



(a)



(b)

Fig. 11. Horizontal loads acting on spandrel beam.

drel members being designed for only two-thirds of the applied shear and torsion load in the end regions, and the concentrated influence of torsion equilibrium connections at the beam end being neglected.

Horizontal Loads

Spandrel beams, as used in parking garages, can be required to restrain automobiles which result in horizontal impact loads as demonstrated in Fig. 11. Horizontal loads to spandrel beams can be applied at any location along the beam's span where " a " of Fig. 11a can vary from zero to L . Both load supporting spandrels and non-load supporting spandrels can be subjected to these horizontal loads.

Fig. 11b illustrates the "cantilever" method of resisting horizontal loads which requires the floor diaphragm (force H_1) and the beam's haunch acting as a horizontal beam (force H_2) to be the load resisting elements. Alternately, horizontal loads can be resisted by the spandrel beam acting as a simple span between its end supports providing that torsional equilibrium is maintained by the required number of connections, or by the beam having adequate torsional strength to transmit the torsion loading to the end torsion equilibrium connections.

Beam End Connections

Spandrel beam end connections which induce forces into the beam can, for the sake of simplicity, be divided into three types. The first type is that associated with the beam's overall torsional equilibrium, the second type deals with "corbel" support behavior, and the third pertains to dapped end support.

The H forces applied by overall beam torsion equilibrium end connections are shown in Fig. 12. The magnitude of the horizontal force couple H providing equilibrium results from all the loads

acting upon the beam, not just those located " $d/2$ " or " d " beyond the end support reaction. Typically, the top H force of the couple is developed by flexural behavior of the beam web. The combination of the torsion equilibrium web flexural stresses, shear stresses, and internal torsion stresses results in the 45-deg cracking repeatedly observed in spandrel beams as illustrated in Fig. 12. Also, the 45-deg cracking is affected by ledge concentrated P loads located near the beam's end inducing additional web stresses.

A different force pattern results when the spandrel end reaction acts upon the ledge. Overall torsion equilibrium of the spandrel beam is achieved by the reaction force R aligning with the applied

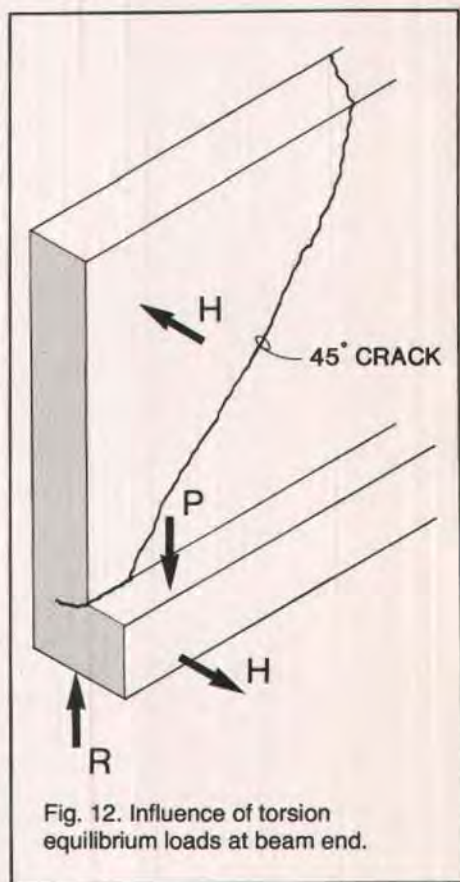


Fig. 12. Influence of torsion equilibrium loads at beam end.

ledge concentrated loads P (neglecting the beam weight), or being beyond the load P . The projecting beam ledge acts like an "upside down" corbel, as shown in Fig. 13, and the projecting ledge must be treated as a corbel if the applied end forces are to be properly considered.

The ends of spandrel beams are sometimes dapped. When daps exist, and the applied concentrated ledge loads P do not align with the reaction R , the forces and stresses resulting from the combined action of the dap and the equilibrium forces (Fig. 12) require complete understanding by the designer. Fig. 14 illustrates the combined action of all forces when a spandrel end support dap is present and the cracking which can develop.

End connection forces applied to simple span spandrel beams also can result from forces necessary to achieve column equilibrium and concrete volume change deformations. However, these factors are more appropriately discussed elsewhere in this paper.

Spandrel Ledges

The ledge, or haunch, of a spandrel beam is the usual mechanism for transfer of the applied concentrated loads to the beam web where the web in turn transmits these concentrated loads to the spandrel support reaction. The ledge transfers load to the beam web via flexure, direct shear, punching shear, and web direct tension.

Two flexure behavior paths exist for transfer of ledge loads to the spandrel web. As shown in Fig. 15, one path is at the vertical interface of the ledge and beam web while the other is at a horizontal plane through the spandrel web in line with the top surface of the ledge. The forces P and N not only induce flexure but also create a state of direct tension. Both force paths must be accounted for.

The dominant shear transfer mode of ledge loads to the beam web is by

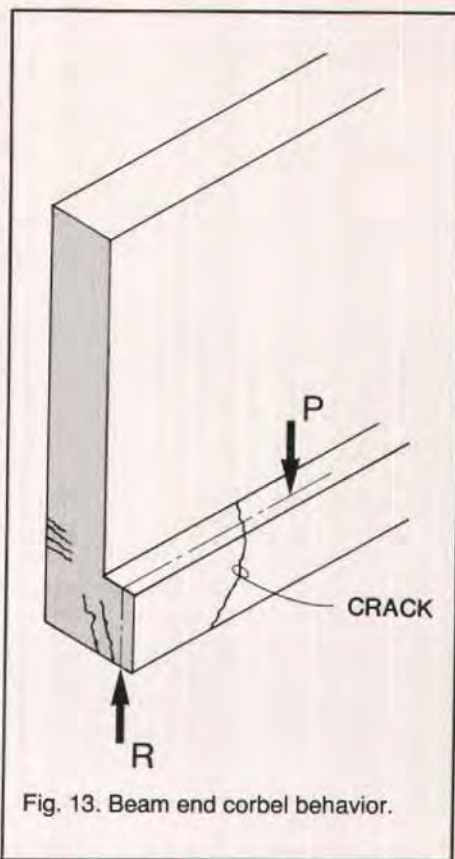
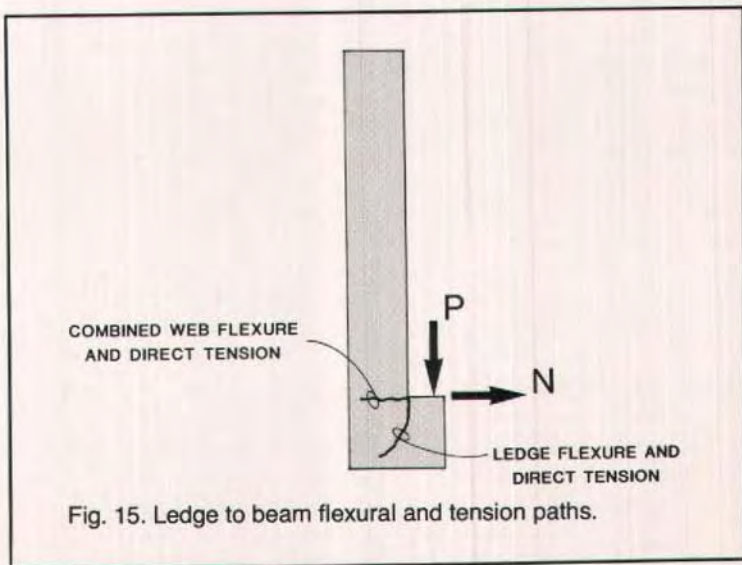
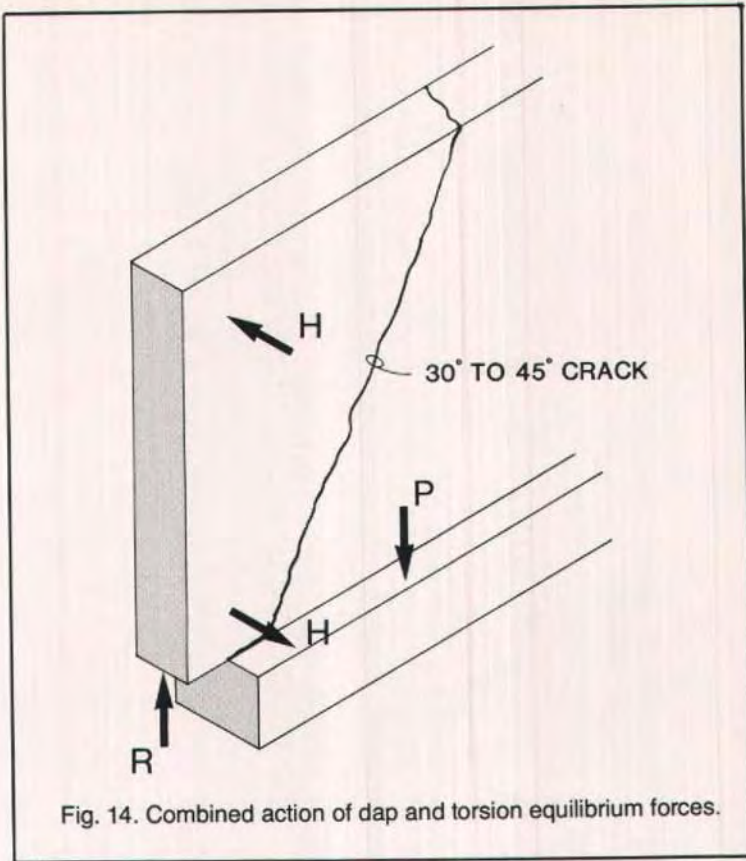


Fig. 13. Beam end corbel behavior.

punching shear which also could be considered as "upside down" dap shear. The location of a concentrated load along the beam's ledge influences the ledge's ability to transmit load generated forces. Fig. 16 illustrates the punching shear transfer of ledge loads to the web for loads applied near the spandrel's end and away from the end.

Volume Change Forces

Volume change forces resulting from restraint of concrete deformations caused by shrinkage, creep, and temperature can occur at the beam's supporting ledge, all connections, and the beam end bearing support. The volume change forces can be axial or rotational as reflected in Fig. 17.



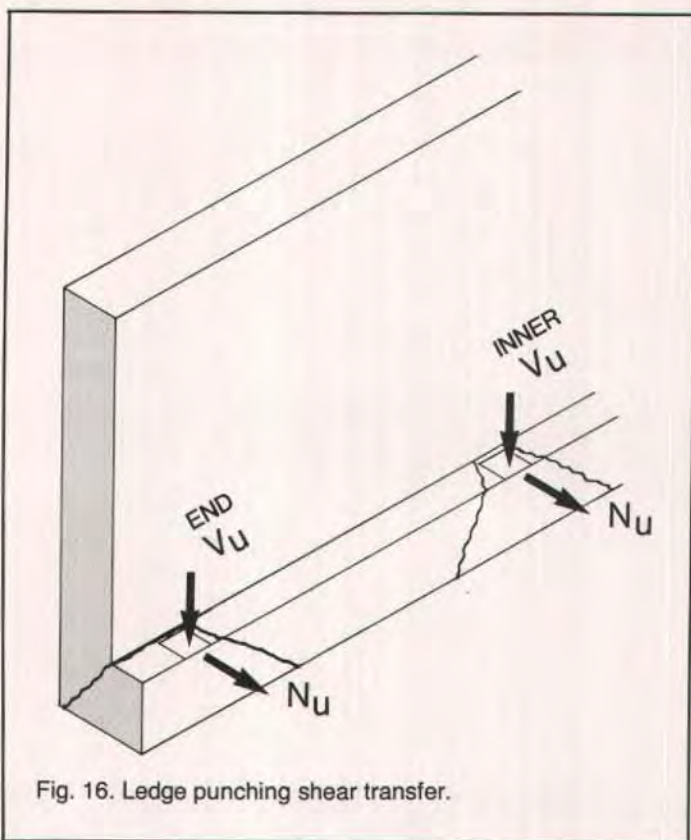


Fig. 16. Ledge punching shear transfer.

Axial volume change forces producing tension in members generally exist, and their magnitude depends on the rigidity of restraint to the volume change movements. For example, welded connections or hard high friction connections can develop large N_u forces whereas members joining one another through bearing pads (soft connections) can result in only minimal N_u forces. The N_u forces acting on spandrel beams can control connection designs, and depending on their magnitude can materially reduce the beam's shear and torsion strength if the N_u force acts parallel to the beam's length. Another factor influencing axial volume change forces in a spandrel is the location of the beam within the building frame horizontally and vertically in addition to the location

of the frame's stiffness center position (position where horizontal volume change movements are zero).

Sun induced temperature differences between the top and bottom of a spandrel beam, where the top surface temperature is greater than the bottom, can create a positive end moment if the beam's end connections are actually rigid. The positive end moment is developed by a combination of horizontal and vertical force couples as shown in Fig. 17b. However, the forces of the couple typically can be neglected for spandrels since in reality they are small because: minute deformations of the connections themselves relieve the rotational restraint; restraint to expansion reduces the horizontal couple force; restraint of shrinkage and creep contrac-

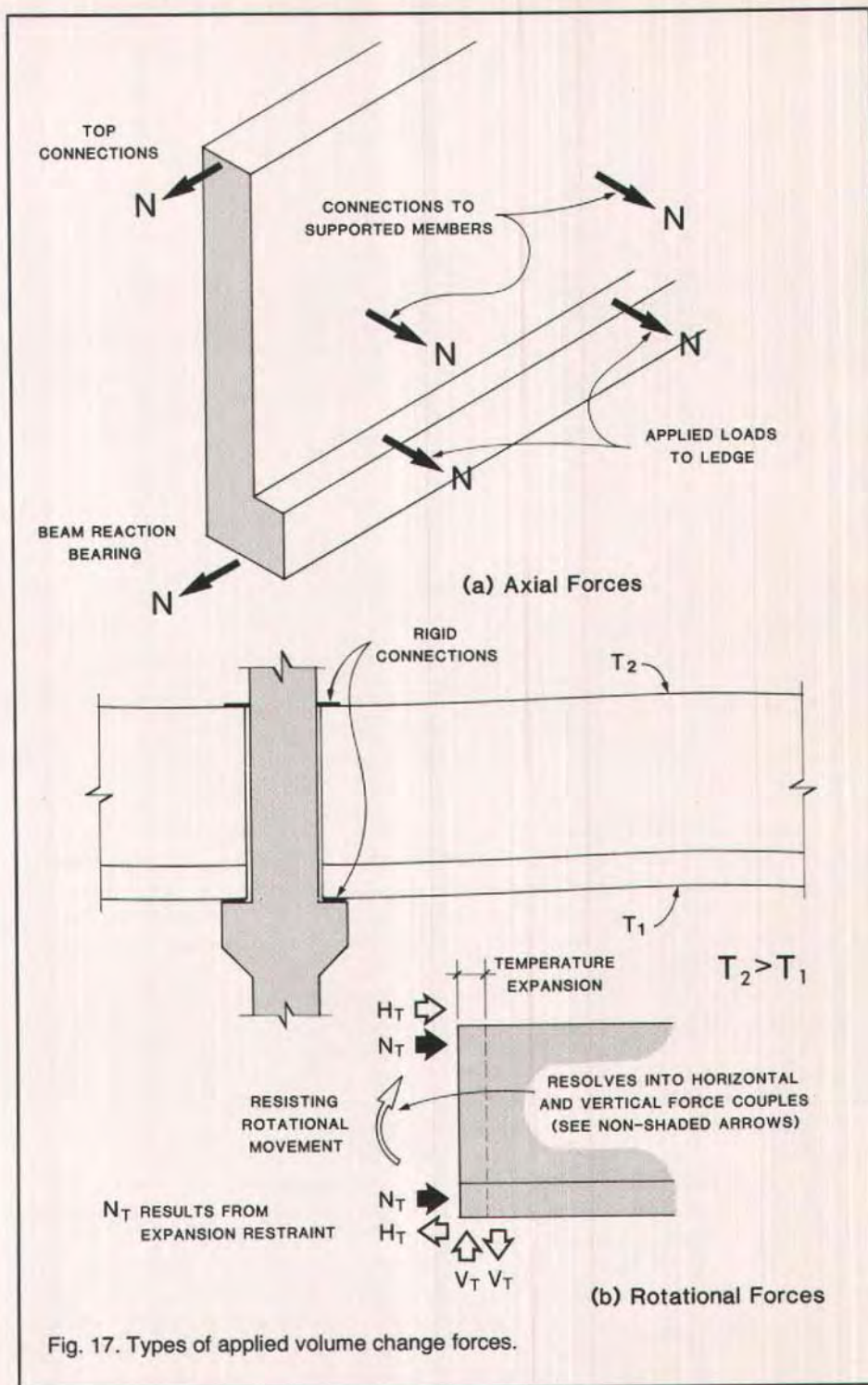


Fig. 17. Types of applied volume change forces.

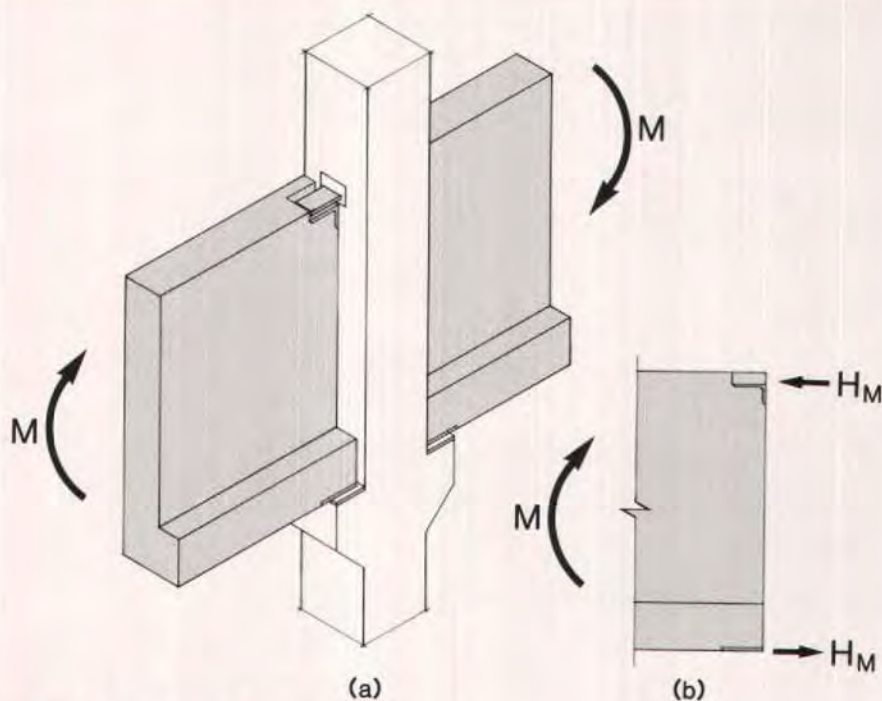


Fig. 18. Moment frame connections.

tion produces rotations opposite to the temperature end rotations; and, for narrow width beam tops, the internal top to bottom temperature variations are small resulting in very minor beam axial and rotational deformations.

Normally, temperature camber influence is of greatest concern to members having a wide top flange width compared to its bottom flange such as double or single tee type beam cross sections when rigid end connections exist. Typically (in northern climates), sun induced temperatures result in the flange average temperature being 30 deg F (17 deg C) greater than the web temperature.

Frame Moment Forces

Spandrel beams can serve as members

of a rigid frame although this is not a common application. When spandrels are part of a moment frame, connections at the beam's end are used to develop horizontal couple forces as shown in Fig. 18. The horizontal couple forces can be of varying magnitude and direction depending upon the mode of the frame's sway, deformations, and type of lateral load applied. The moment frame horizontal couple forces are additive or subtractive to the other forces at the spandrel's end(s) when determining overall design forces. If the moment frame spandrel beam also supports gravity loads, the gravity loads produce additional moments which must be combined with other frame moments. Gravity dead loads may or may not cause end moments, depending upon when frame connections are made.

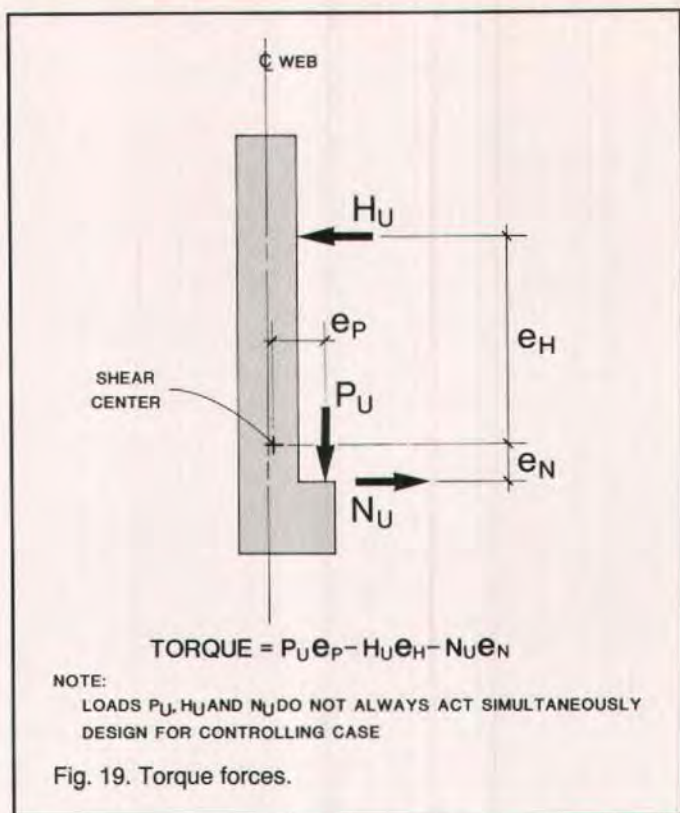


Fig. 19. Torque forces.

GENERAL DESIGN REQUIREMENTS

Spandrel beam design requires consideration of how all the various applied loadings are transmitted from their point of application to the beam and then subsequently to the structural element(s) supporting the spandrel itself. Basic design requirements, exclusive of the connections to columns or other supporting structural elements, are:

- Internal torsion and shear
- Beam end torsion
- Ledge attachment to the web
- Ledge load transfer
- Web flexure resulting from torsion equilibrium
- Ledge acting as a corbel at beam end reaction
- Beam flexure

Internal Torsion and Shear

Spandrel beam torsion results when applied horizontal and vertical loads do not pass through the beam's shear center. The resulting torsion to the beam, at any cross section, is the sum of the torques (shear force times distance from the shear center) acting at that cross section. Moreover, it is possible that the loads acting on the spandrel can vary from the time of erection to when all time-dependent volume change loads act. Each loading case requires evaluation to determine which controls the design.

Fig. 19 shows the loads applied to a spandrel beam eccentric to the shear center. The determination of the shear center is provided by Fig. 20⁴ for a homogeneous uncracked section. Once the applied torsion and shear are known, the

internal torsion and shear reinforcement can be determined following ACI 318³ requirements for reinforced members or employing other relationships^{3,5} for prestressed members. If only vertical loads are applied to the spandrel, the shear center can quite conservatively be assumed to align with the beam's web vertical centerline.

Beam End Torsion

Beam end torsion is defined as the torsion at the beam's end, within the distance " d " or " $d/2$," resulting from the torsion equilibrium end connections. Typically, beam end torsion created by top and bottom connections is characterized by a single crack, inclined at about 45 deg, having a nominal width of 0.015 in. (0.38 mm) or greater. The 45-deg crack results from flexure in the spandrel's web induced by the beam top H_u connection force (see Fig. 21a) required to maintain overall torsion equilibrium. The magnitude of the H_u forces is that required to resist all loads eccentric to the shear center, and not just those loads at and beyond " d " or " $d/2$ " from the beam's vertical end support. The end of the beam length required to transfer the H_u equilibrium forces to internal torsion behavior can be considered to be within the distance " d " from the beam's end.

Beam end reinforcement to resist the web flexure caused by the H_u forces can be supplied by horizontal and vertical bars. The orthogonal bars consist of steel areas A_l , A_v , and A_t (according to ACI 318-77) located on the ledge side of the beam's web (see Fig. 21b). The end reinforcement effective in providing the necessary 45-deg A_w steel of Fig. 21d is given by Fig. 21c. The web A_w reinforcing steel required for ultimate strength is:

$$A_w = \frac{H_u e_w}{\phi d_w f_y} \quad (1)$$

where all dimensions are in inches, the

force is in lbs or kips, the stress is in psi or ksi, and $\phi = 0.85$.

Note that ϕ equal to 0.85 results from using $\phi = 0.90$ for flexure and $j_u = 0.94$ such that the denominator for Eq. (1) is:

$$\phi j_u d_w f_y = 0.9 (0.94) d_w f_y = 0.85 d_w f_y$$

The spacing of the A_w reinforcement, horizontally and vertically, should not be greater than 12 in. (305 mm).

If the amounts of the vertical stirrups and horizontal bars distributed about the spandrel's perimeter differ, the quantity of A_w available depends on the lesser amount of A_{wv} and A_{wl} (see Fig. 21b):

$$A_w = \frac{2 (\Sigma A_{wv})}{\sqrt{2}} \quad (2)$$

or

$$A_w = \frac{2 (\Sigma A_{wl})}{\sqrt{2}} \quad (3)$$

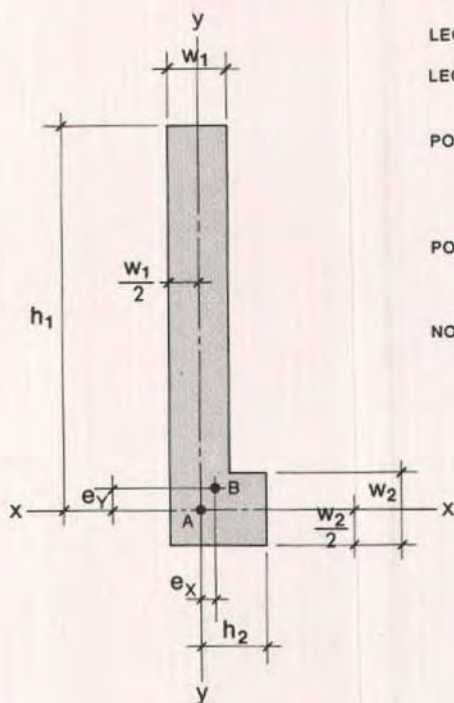
whichever results in the smaller value for A_w . The ΣA_{wv} or ΣA_{wl} represents the total amount of reinforcing steel within the effective dimensions defined by Fig. 21c. Anchorage of the A_{wv} and A_{wl} reinforcing bars is important. Both ends of the A_{wv} steel must be anchored by hooks or closed stirrups. The A_{wl} bars require hooking at the end of the beam or a "U" shape for proper anchorage.

The vertical and horizontal reinforcement comprising the A_w steel at the beam's end is in addition to any other reinforcement necessary to resist shear since the A_w and shear reinforcement share a common crack plane.

Another type of special beam end torsion is discussed in a following section titled "Corbel End Behavior." This special case, as for the above, requires the beam's end to be designed to resist the total torsion acting at the beam's support.

Ledge Attachment

Attachment of the spandrel beam



LEG 1 = RECTANGLE $w_1 h_1$

LEG 2 = RECTANGLE $w_2 h_2$

POINT A = INTERSECTION OF VERTICAL &
HORIZONTAL LEG CENTERLINES
($y-y$ & $x-x$)

POINT B = SHEAR CENTER

NOTE: IF w_1 AND w_2 SMALL, $e_X = e_Y = 0$.
AND SHEAR CENTER B IS AT A.
CONSERVATIVE TO SELECT SHEAR
CENTER AT POINT A.

CALCULATION OF e_X

$$e_X = \frac{h_2}{2} \left(\frac{I_2}{I_1 + I_2} \right) = \frac{h_2}{2} \left[\frac{h_2 w_2^3 / 12}{w_1 h_1^3 / 12 + h_2 w_2^3 / 12} \right]$$

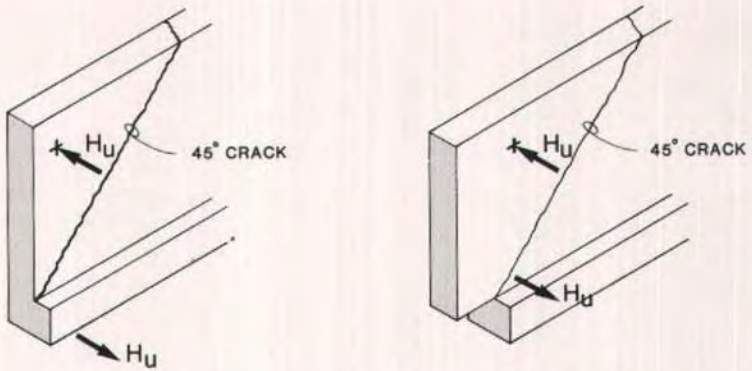
$$e_X = \frac{h_2}{2} \left[\frac{h_2 w_2^3}{w_1 h_1^3 + h_2 w_2^3} \right]$$

CALCULATION OF e_Y

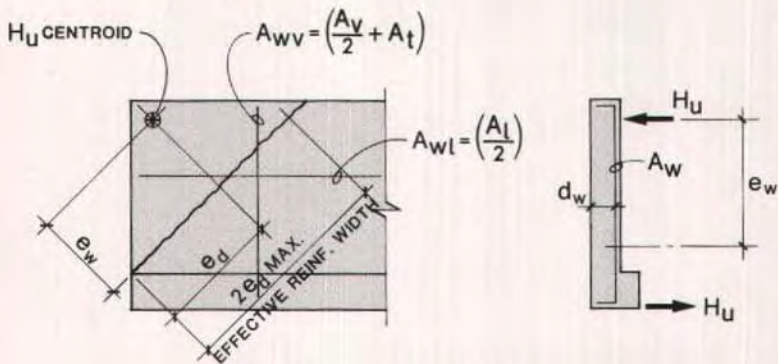
$$e_Y = \frac{h_1}{2} \left(\frac{I_1}{I_1 + I_2} \right) = \frac{h_1}{2} \left[\frac{h_1 w_1^3 / 12}{h_1 w_1^3 / 12 + w_2 h_2^3 / 12} \right]$$

$$e_Y = \frac{h_1}{2} \left[\frac{h_1 w_1^3}{h_1 w_1^3 + w_2 h_2^3} \right]$$

Fig. 20. Calculation of spandrel beam shear center.

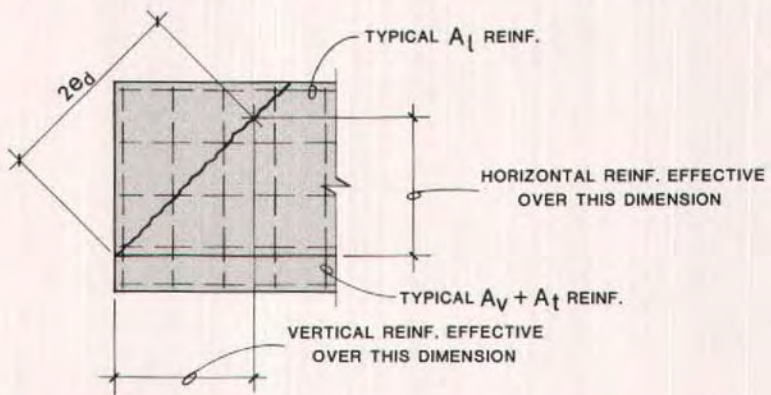


(a) End Web Cracking



(b) End Torsion Nomenclature

(d) End A_w



(c) Orthogonal End Reinforcement

Fig. 21. Beam end torsion equilibrium reinforcement.

ledge to the web can be either by the strength of plain concrete or by reinforcing steel depending upon the beam dimensions, concrete strength, and magnitude of the ledge load. The position of the load V_u also can have an influence on the ledge to web capacity. Accordingly, considering the fabrication tolerances of the framing members, interfacing erection tolerances, and deformation characteristics of the ledge supported member, the position of V_u (see Fig. 22) should be located at $\frac{3}{4} l_p$.

The ledge to web horizontal attachment is considered similar to the behavior of two rigid bodies where separation would occur along the entire length of the beam's web on the attachment plane of Fig. 22a. Two separate conditions can exist as shown by Fig. 22a, and concern an end load (near the beam's end) or an inner load (away from the beam's end). The concentrated loads acting are shown in Fig. 22, but the ledge attachment also must consider uniform loads. Uniform loads would produce V_u 's equal to the uniform load times the ledge length being examined.

The ultimate attachment strength of a non-reinforced ledge depends on the tensile strength of the concrete. The point of maximum ultimate tension is A (see Fig. 22b). The maximum stress f_t results from a combination of direct tension and flexure, and is:

$$f_t = \frac{V_u}{b_w m} + \frac{6 V_u (b_w/2 + \frac{3}{4} l_p)}{b_w^2 m} \quad (4)$$

where s and d_e have maximum values in accordance with Fig. 22. All dimensional units are in inches and the force is in lbs or kips. Note that:

$m = s$ for inner concentrated loads

$m = d_e + s/2$ for end concentrated loads

$m =$ length selected for analysis of uniform load and $V_u = m$ times unit uniform load

$f_t = 3\lambda \sqrt{f'_c}$ psi tension maximum at

ultimate where $\lambda = 1$ for normal weight concrete and 0.85 for sand lightweight concrete

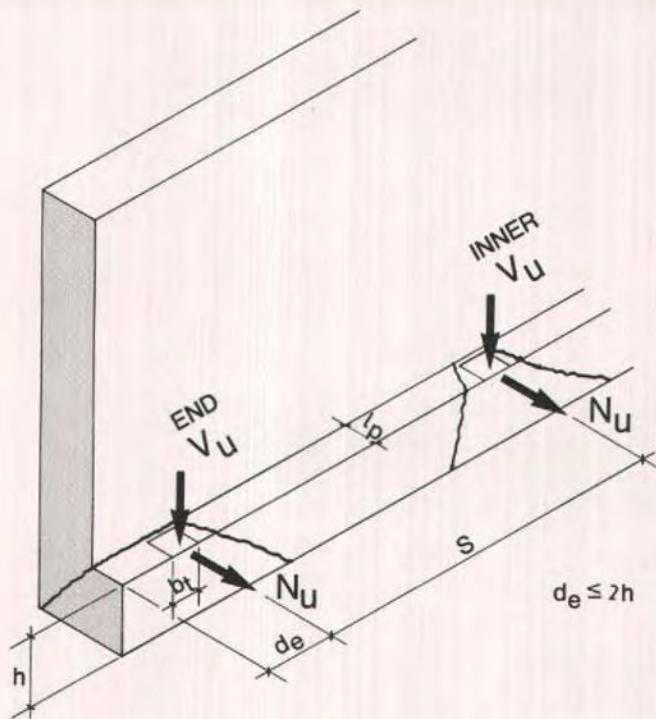
The f_t maximum value at ultimate of $3\lambda \sqrt{f'_c}$ for f_t is derived from the traditional concrete cone punching shear direct tension limit of $4 \sqrt{f'_c}$ using a phi factor of 0.75 and λ to account for the concrete unit weight. Moreover, considering the nominal reinforcements which usually exist in both the beam web and ledge, the value of $3\lambda \sqrt{f'_c}$ is a lower bound value when ACI 318-77 or ACI 322-72⁶ allows $f_t = \phi 5 \sqrt{f'_c}$ ($\phi = 0.65$) for unreinforced concrete flexure at ultimate. If f_t exceeds $3\lambda \sqrt{f'_c}$, attachment reinforcement is required.

Reinforcement for attachment of the spandrel ledge to the beam's web is shown in Fig. 22c, along with the parameters for determining the amount of reinforcing steel required. Typically, the depth of the ultimate stress block is less than 1 in. (25.4 mm) for usual design conditions. Assuming the stress block centroid is at 0.5 in. (12.7 mm) ($a/2$) per Fig. 22c, and summing moments about this centroid, the reinforcement for attaching the ledge to the web is:

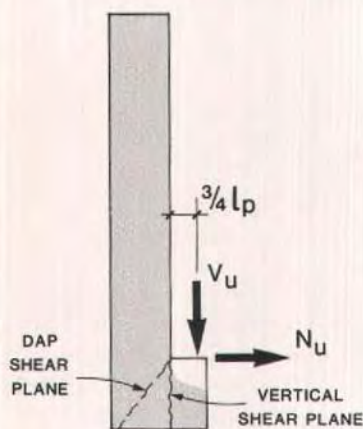
$$A_s = \frac{V_u (b_w + \frac{3}{4} l_p - \frac{1}{2})}{\phi f_y (b_w - d_s - \frac{1}{2})} \quad (5)$$

where $\phi = 0.85$, all dimensional units are in inches, the force is in lbs or kips, the stress is in psi or ksi, and the ledge to web reinforcement is uniformly distributed over the length m as previously defined.

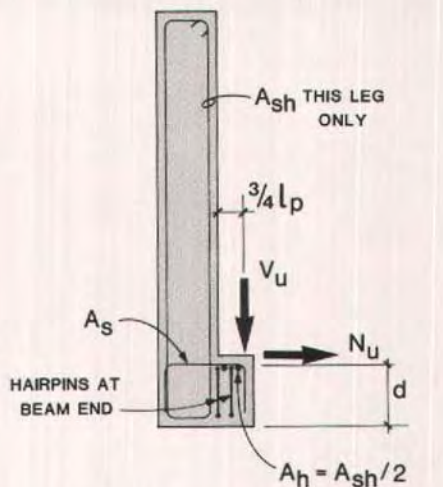
The A_s reinforcement of Eq. (5) can in part or whole be supplied by web stirrups in the beam. The ledge attachment reinforcement usually is not additive to shear and torsion reinforcing because the A_s web steel reinforces a different crack plane. The amount of beam web reinforcing will be controlled by the greater of the requirements for combined shear and torsion or ledge attachment. The spacing of A_s bars should not exceed 18 in. (457 mm).



(a) Punching Shear Transfer



(b) Dap Shear



(c) Ledge Transfer Reinforcing

Fig. 22. Beam ledge attachment for non-reinforced and reinforced sections.

Ledge Load Transfer

The spandrel beam's ledge transfers uniform and concentrated loads to the web by shear and flexure. The engineering procedures presented for transfer of the ledge loads are based on the PCI Design Handbook,¹ with some variations, and are restated to keep this presentation all inclusive for spandrel beam design.

The ledge load transfer must satisfy concrete punching shear if special shear reinforcement is not to be used. Fig. 23a portrays concentrated loads applied to the ledge and the ultimate punching shear pattern. The ultimate V_u capacity for inner ledge loadings and end loadings when d_e equals or exceeds $2h$ is:

For $s > b_t + 2h$

$$V_u = 3\phi h\lambda\sqrt{f'_c} (2l_p + b_t + h) \quad (6)$$

For $s \leq b_t + 2h$

$$V_u = \phi sh\lambda\sqrt{f'_c} \quad (7)$$

where $\phi = 0.85$, all dimension units are in inches, f'_c is in psi, and λ is as previously defined.

The ultimate V_u capacity for end ledge loading, where d_e is less than $2h$, is:

For $s > b_t + 2h$

$$V_u = \phi h\lambda\sqrt{f'_c} [2l_p + (b_t + h)/2 + d_e] \quad (8)$$

For $s \leq b_t + 2h$

$$V_u = \phi h\lambda\sqrt{f'_c} (d_e + s/2) \quad (9)$$

where the notation is the same as for Eqs. (6) and (7).

As for closely spaced concentrated loads, the ultimate capacity for uniform loads applied to the ledge is the same as given by Eq. (7), which for an s of 1 ft (0.305 m) provides:

$$V_u = \phi 12h\lambda\sqrt{f'_c} \quad (10)$$

Eqs. (7) through (10) use an ultimate shear stress of $1\sqrt{f'_c}$ on the vertical shear plane shown in Fig. 23b. Unpublished

tests by the author have indicated that the ultimate dap unreinforced concrete shear stress capacity, when using the vertical shear plane as the measure, is $1.2\lambda\sqrt{f'_c}$ for normal weight and sand lightweight concretes having an $f'_c = 5000$ psi (34 MPa) and the load applied on the ledge at $3/4l_p$. A value of $3\sqrt{f'_c}$ is used on all shear planes in Eq. (6) while Eq. (8) uses a value of $2\sqrt{f'_c}$ on the projecting ledge shear area hl_p .

Applied V_u forces in excess of the unreinforced V_u capacities determined by Eqs. (6) through (10) require dap reinforcement, A_{sh} (see Fig. 23c). The necessary A_{sh} to resist the applied V_u is:

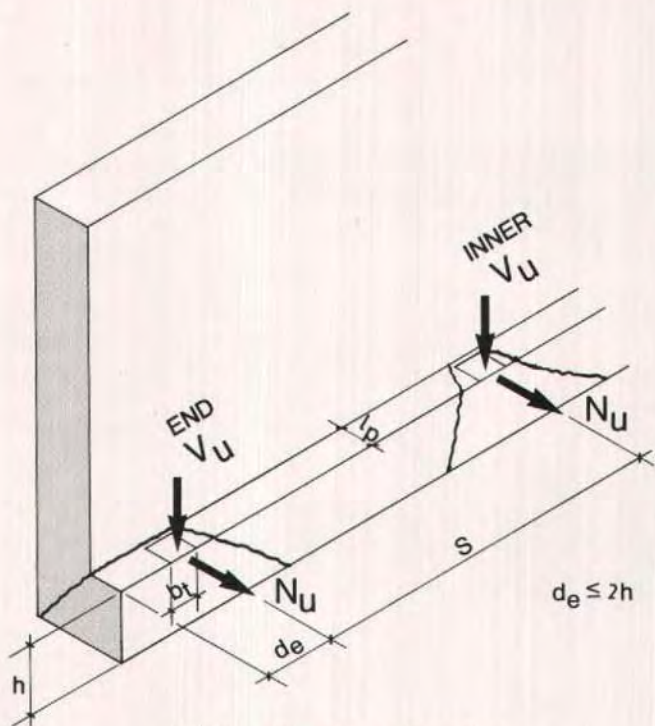
$$A_{sh} = \frac{V_u}{\phi f_y} \quad (11)$$

where $\phi = 0.85$, the V_u force is in lbs or kips, and the stress f_y is in psi or ksi.

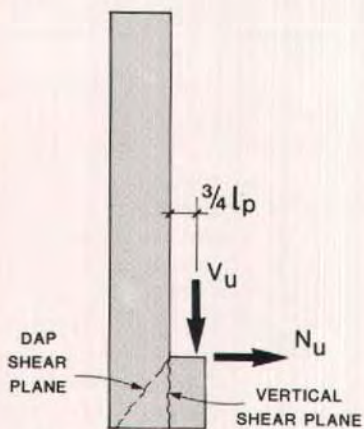
The ultimate shearing stress on the ledge vertical shear plane when A_{sh} steel is used (see Fig. 23b) should not exceed $10\lambda\sqrt{f'_c}$. The vertical shear plane area for concentrated loads spaced further apart than $b_t + 2h$ is $h(b_t + h)$. Fig. 23 can be used as a guide in determining the vertical shear plane area for other loadings.

The A_{sh} reinforcement determined from Eq. (11) should be totally with $b_t + h$ for inner and end loads, $d_e + (b_t + h)/4$ for end loads when d_e is less than $(b_t + h)/2$, and within s for closely spaced concentrated loads or uniform loads unless a greater spacing can be justified by the longitudinal reinforcement in the spandrel ledge laterally distributing the concentrated load.

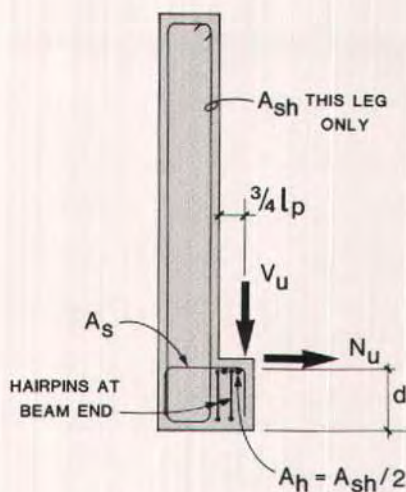
Longitudinal reinforcement, A_h , in the beam's ledge should be, at a minimum, equal to $A_{sh}/2$ to insure all A_{sh} reinforcement over its distribution width is engaged. The longitudinal reinforcement distributes concentrated ledge loads along the ledge by both dowel shear and flexure when the reinforcement is located near the ledge top as shown in Fig. 23c.



(a) Punching Shear Transfer



(b) Dap Shear



(c) Ledge Transfer Reinforcing

Fig. 23. Ledge load transfer.

The A_h at the end of the beam should confine the end of ledge, and typically have a hairpin configuration which laps to the continuous ledge A_h . The steel required by Eq. (11) is not additive to the stirrups required to resist web shear and torsion since it basically reinforces a different crack plane. However, when A_{sh} is required, more stirrups may be necessary at the concentrated V_u locations so the total steel area provides the calculated A_{sh} distributed over the lengths previously discussed. If A_{sh} is not provided by additional stirrups, then carefully anchored separate reinforcement on the ledge side of the web should be used.

Ledge flexural reinforcement is required to resist the applied V_u and N_u loads. The A_s steel (see Fig. 23c) can be selected from:

$$A_s = \frac{1}{\phi f_y} \left[V_u \left(\frac{3 L_p}{4 d} \right) + N_u \left(\frac{h}{d} \right) \right] \quad (12)$$

where $\phi = 0.85$, forces are in lbs or kips, dimensions are in inches, and the f_y stress is in psi or ksi.

The A_s reinforcing steel using the bent configuration of Fig. 23c does not provide bearing confinement steel. If reinforced bearing confinement steel is necessary, special reinforcement in accordance with the PCI Design Handbook¹ is required.

The A_s reinforcing of Eq. (12) should be centered on the load and distributed within the ledge over the same distance as the A_{sh} reinforcing bars for inner and end loads, but not exceeding $b_l + h$. Bar spacing should not exceed h nor 18 in. (457 mm). It is suggested that #3 bars at the maximum spacing be provided as a minimum for ledge flexural reinforcement.

Web Flexure

Two instances of web flexure can develop if the spandrel beam's overall torsion equilibrium is generated by the beam web acting against the top of the

members it supports and bottom connections at the spandrel's end vertical reaction. This condition is illustrated in Fig. 24 where there is no top end connection to provide the necessary overall torsion equilibrium. A similar loading condition can occur when horizontal loads are applied to the beam web (see Fig. 11b), except that the forces have a direction opposite of those resulting from vertical load torsion equilibrium.

Fig. 24a presents a typical cross section away from the beam ends showing the w_{tu} force developed by the topping acting against the web. The distribution of the unit torsion load w_{tu} against the web is given by Fig. 24b for a spandrel beam having equal tee stem reactions and uniform stem spacing where w_{tu} is:

$$w_{tu} = \frac{T}{(L/2)h_h} \quad (13)$$

and

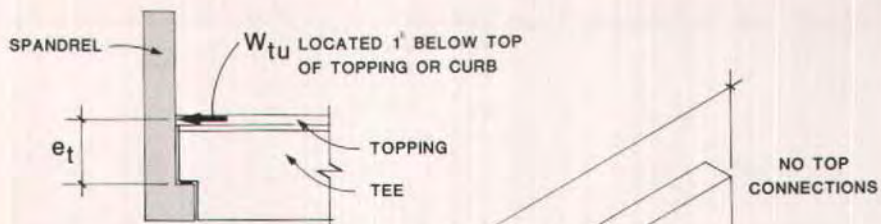
$$H_u = w_{tu}(L/2) \quad (14)$$

Note that T in Eq. (13) represents the total overturning torque for half the beam, and the other variables are defined in Fig. 24b.

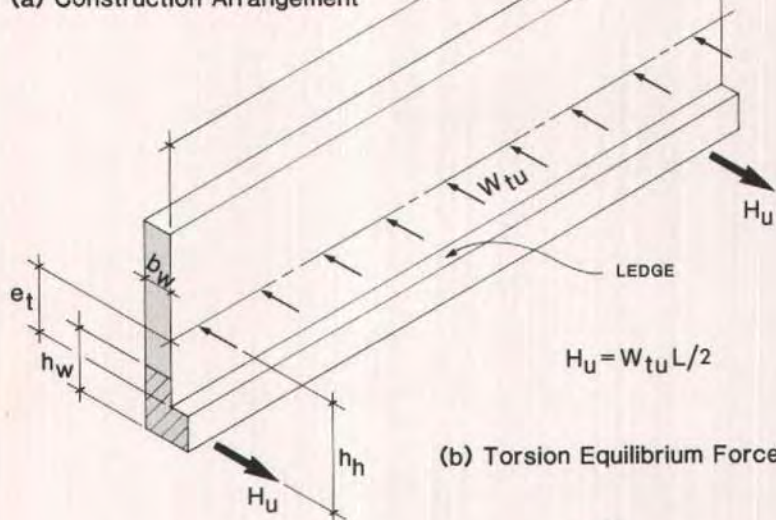
The w_{tu} force acting against the web can be transferred by web flexure (see Fig. 24c) to the lower web portion of the beam and then by the lower web portion to the beam's ends. The height of the lower web portion h_w is shown in Figs. 24b, 24d, and 24e. The lower web portion h_w transfers w_{tu} to beam ends by horizontal flexure as demonstrated in Fig. 24d. In turn, this horizontal ultimate flexural behavior of the lower web portion can be resisted by reinforcement, which is in addition to that required for vertical flexure (see Fig. 24e).

Depending upon force magnitudes, web reinforcement (see Fig. 24c) may be necessary to resist the web flexure unless:

$$f_w = \frac{w_{tu}(e_v)}{2b_w^2} \leq 3\lambda\sqrt{f'_c} \quad (15)$$



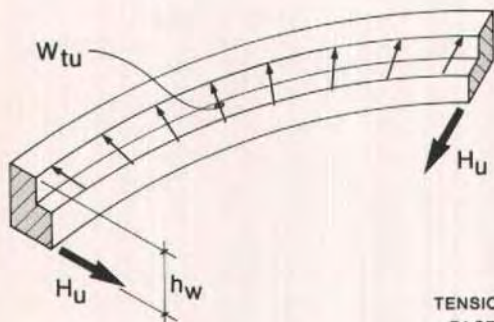
(a) Construction Arrangement



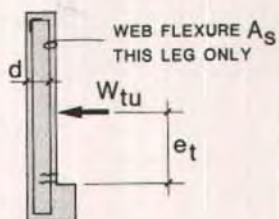
(b) Torsion Equilibrium Forces

$$h_w \leq 2b_w + h$$

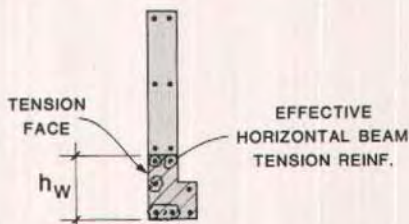
$$\leq e_t / 2$$



(d) Lower Web Flexure
(SIMPLE SPAN CASE)



(c) W_{tu} Web Flexure



(e) Ledge Horizontal Beam

Fig. 24. Torsion equilibrium web flexure.

where w_{tu} is in lbs per linear foot at ultimate, e_t is in inches, b_w is in inches, and λ and f'_c are as previously defined.

If f_w exceeds $3\lambda\sqrt{f'_c}$, the web reinforcement A_w per foot of length is:

$$A_w = \frac{w_{tu}(e_t)}{\phi d f_w} \quad (16)$$

where all dimensional units are in inches, w_{tu} at ultimate is in lbs per linear foot, ϕ is 0.85, and f_w is in psi.

The A_w determined by Eq. (16) is additive to the ledge attachment reinforcement, calculated by Eq. (5) and/or the A_{sh} steel resulting from Eq. (11) due to common crack planes, but is not additive to the web shear reinforcement which reinforces a different crack plane. Usual web torsion steel would not be required if the spandrel beam's torsional equilibrium is provided by the behavior depicted in Fig. 24. The spacing of the web A_w should not exceed 18 in. (457 mm) nor $3b_w$.

In actual practice, torsional equilibrium can be provided by a modified behavior to that shown in Fig. 24. This modified behavior follows when the spandrel beam has adequate torsion reinforcement, proper bottom H_u overall torsion equilibrium connections (see Fig. 24b), and the top connection is at the topping level (see Fig. 24a), but only a limited portion of the topping at the beam end is involved in developing H_u .

Using the topping to provide one of the horizontal force components for overall beam torsion equilibrium requires special attention to the connection of the column to the topping. This consideration is discussed more fully in a following part of the paper.

Corbel End Behavior

When the end support reaction of a spandrel beam aligns with the applied ledge loads, the ledge acts like an upside down corbel as previously discussed for Fig. 13. The upside down ledge corbel can be designed to support

the end reaction by following the procedures of the PCI Design Handbook¹ for corbels with some modifications.

Fig. 25 shows the usual variables and parameters affecting corbel end behavior. In Fig. 25, the end corbel reinforcement is given by Fig. 25a for internal reinforcing bars and by Fig. 25b which represents a combination of a steel bearing plate providing A_w and the internal reinforcement. An elevation of the end corbel reinforcement is illustrated in Fig. 25c. The corbel A_{s1} steel at the beam bottom is determined from:

$$A_{s1} = \frac{3 V_u l_p}{\phi 4 f_y d_1} \quad (17)$$

or

$$A_{s1} = \frac{V_u^2}{\phi f_y 2 \lambda^2 b_1 d_1} \quad (18)$$

whichever is the greater, but a minimum of at least:

$$A_{s1} = \frac{0.08 b_1 d_1}{f_y} \quad (19)$$

where in the above equations A_{s1} is in square inches, λ is as previously defined, ϕ is 0.85, f_y is in ksi, V_u is in kips, and all dimensions are in inches.

The additional ledge reinforcement A_{vh} , using identical notation, is:

$$A_{vh} = \frac{V_u^2}{\phi f_y 4 \lambda^2 b_1 d_1} \quad (20)$$

or a minimum of at least:

$$A_{vh} = \frac{0.04 b_1 d_1}{f_y} \quad (21)$$

Reinforcement A_{s2} in the beam web at the corbel end (see Fig. 25a) is required to deliver the corbel moment. This reinforcing steel can be approximately selected using:

$$A_{s2} = \frac{d_1}{d_2} (A_{s1}) \quad (22)$$

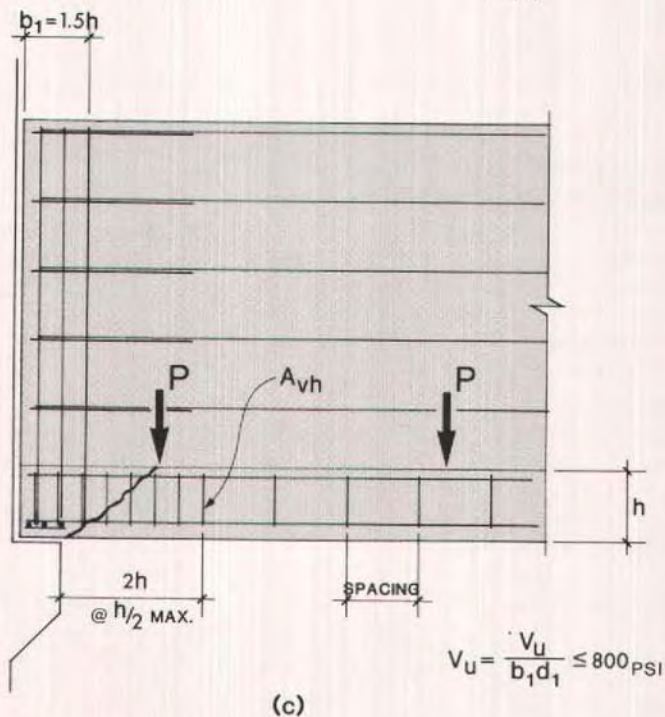
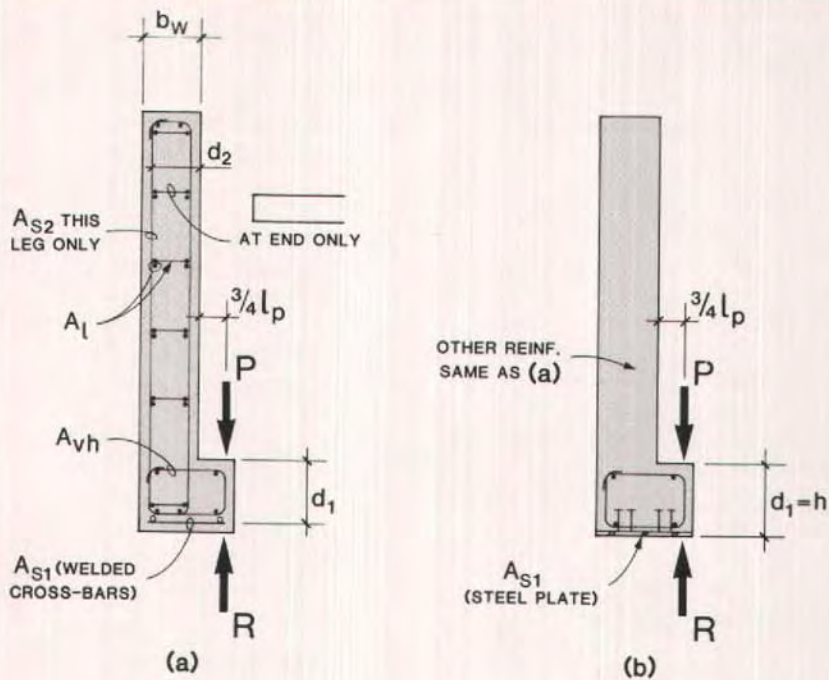


Fig. 25. Corbel end reinforcement.

where all variables are in the same units.

Proper distribution of the reinforcement (Fig. 25) is important. For example, A_{s1} and A_{s2} should be placed within $1.5h$. The required A_{s3} should likewise be located within $1.5h$ but also should be distributed from the beam end to a distance of $2h$ beyond the beam reaction support, and should not be spaced greater than $h/2$. Attention must be paid to all steel anchorages to insure that the bars will develop their strength at critical sections. The steel plate in Fig. 25b, if used to provide A_{s1} , needs the correct number of headed studs for anchorage to develop the plate strength.

Experience indicates that the unit end corbel shear stress at ultimate should not generally exceed 800 psi (5.5 MPa). Also, the spandrel web beyond $1.5h$ from the beam's end should be designed to resist the total torsion developed by the end reaction acting eccentric to the shear center.

When corbel end behavior is used to provide overall torsion equilibrium, careful evaluation must be given to nonuniform bearing stresses at the ledge face, the bearing pad, and the supporting column member. If bearing pads are used where corbel end behavior provides overall torsion equilibrium, torsional roll of the beam may result from the pad's deformation depending on the type of bearing pad employed.

Beam Flexure

General spandrel beam flexure involves two distinct loading conditions — one at service level and the other at ultimate state. Depending upon the beam's cross-sectional dimensions, and whether or not the spandrel employs reinforcing bars or prestressing strands, the methods of analysis can be basically the same or fundamentally different.

If connections between the spandrel beam and the structural units supported by the spandrel do not prevent torsional

rotations, then it may be necessary to consider the influence of principle axes of inertia relative to service loads. Spandrel beams usually do not have symmetry about either axis. And, if the depth of the beam is shallow, then when determining elastic stresses at service level, for either reinforcing bars or prestressed reinforcement, the orientation of the principle axes possibly can be critical. Fig. 26 presents the concept of principle axes of inertia found in most textbooks on strength of materials.

The influence of principle axes of inertia can typically be neglected for deep spandrel beams. However, for small depth spandrels, and particularly those employing prestressing, principle moments of inertia may require consideration. The orientation of the principle axes is:

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2 I_{xy}}{I_y - I_x} \right) \quad (23)$$

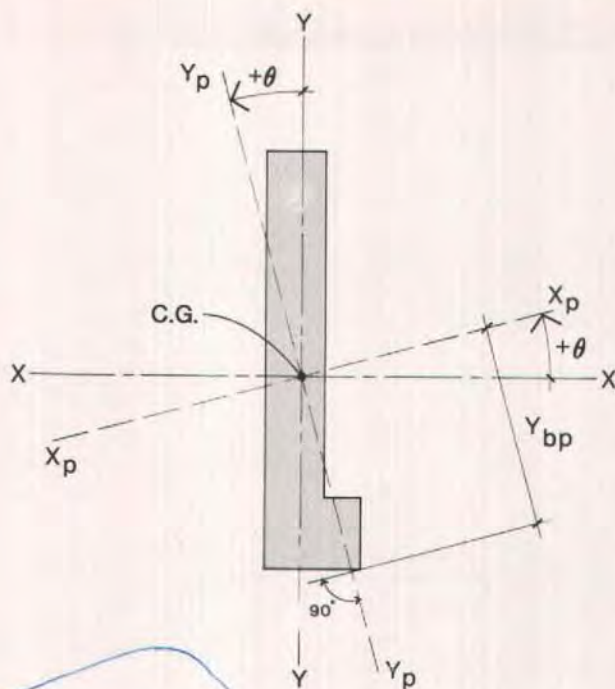
where the angle θ , in degrees, is positive counterclockwise for the dimensional orientation (the ledge to the right side) given in Fig. 26. The principle moments of inertia are:

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2} \right)^2 + I_{xy}^2} \quad (24)$$

and

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2} \right)^2 + I_{xy}^2} \quad (25)$$

Flexural analysis of spandrel beams at ultimate should follow the requirements of ACI 318-77,³ Chapter 10. The analytic procedures can be based upon approximate methods or the specific requirements pertaining to strain compatibility where it is desired to account for all the reinforcements relative to their distributions in both the tension and com-



pal

X_p AND Y_p DEFINE
ORIENTATION OF PRINCIPLE AXES

$$S_{bp} = \frac{I_{MAX.}}{Y_{bp}}$$

Fig. 26. Orientation of principle moments of inertia.

pression zones. When making an ultimate flexural analysis, consideration must be given to those longitudinal reinforcements which are required to provide for torsion and torsional equilibrium, and therefore cannot contribute to the flexural strength.

CONSIDERATIONS FOR CONNECTION DESIGN

Spandrel beam connection design is similar to the design for other precast prestressed concrete connections. All of the PCI requirements^{1,2} apply to connections associated with spandrel beams. The critical aspects of spandrel

connection design are understanding the concepts, accounting for all connection forces, and evaluating the beam's loading history from the time it is first erected through application of in-use service (or ultimate) loads.

This discussion on spandrel connections is intended to supplement the PCI connection design requirements by reviewing the concepts and parameters involved. Discussions on connections herein relate to connection systems, connection materials, connection interfacing, frame moment connections, internal connection reinforcement, and connection loads resulting from equilibrium requirements of the column supporting the spandrel beam.

Connection Systems

Spandrel connection systems can be separated into three groups. These groups are the basic beam-to-column connections, connections between the spandrel and the members supported by the beam, and those connections required at the time of erection.

Some of the more common spandrel beam-to-column arrangements are shown in Fig. 27. The conditions of Figs. 27a and 27b result in non-alignment of applied ledge loads and the column reaction thereby requiring connections to achieve overall torsion equilibrium. The equilibrium connections use inserts and bearing pads to develop the horizontal force couples. The conditions reflected in Figs. 27a and 27b are for parking garages where the curb is also part of the overall connection system and influences the volume change forces induced into the connections, column lateral equilibrium reactions, and requirements for erection connections.

Figs. 27c and 27d depict spandrel end connections where the applied ledge loads and column reactions align. The dap of Fig. 27d uses a modification to the beam end to insure load and reaction alignment. This modification requires careful consideration of the tee leg spacing [5 ft (1.52 m) spacings typically] and layout of the tees, and its use is generally limited to a maximum column dimension of 18 in. (457 mm) for the column face shown in the Fig. 27d elevation.

Spandrel beam-to-column arrangements for a building, shown in Fig. 28, require overall torsion equilibrium connections. The top connections used at the floor and roof levels employ welds requiring special details to limit the magnitude of the volume change forces to the connections. When using top welded torsion equilibrium connections, the magnitude of the volume change force can be limited to the

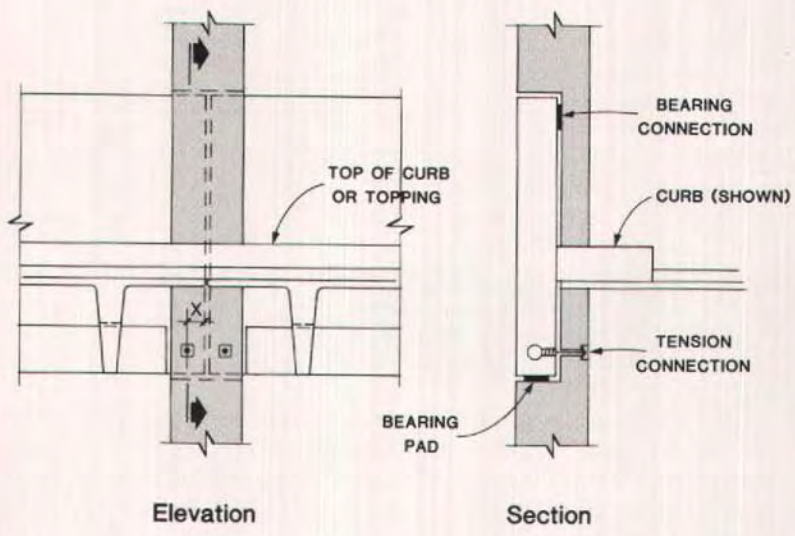
plate's yield stress force by controlling the cross section of the attaching plate and placing welds only at the plates' ends to allow for deformation strain beyond yield. The attachment plate width, plus holes in the plate if required to limit its tension yield capacity, and thickness should only be that necessary to resist the torsion couple shear force. If the building spandrel-to-beam detail is similar to Fig. 27c, torsion equilibrium connections are not required.

In Fig. 28b, the column is shown as projecting to the top of the spandrel beam rather than having the beams bear upon the column top. The projection of the column to the top of the beam is required to achieve the spandrel torsion equilibrium in addition to providing the column top with the necessary horizontal reaction for its lateral equilibrium.

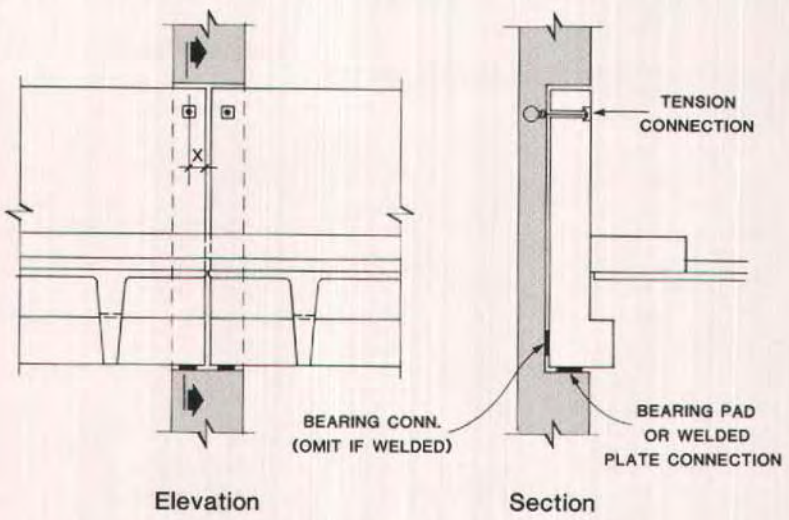
Structural design requirements for in-place conditions, or for temporary erection stability and equilibrium, can dictate the need for connections between the spandrel beam and the members the spandrel supports. An example of this type of spandrel connection is shown in Fig. 29. Regarding the attachment plate connections of Fig. 28, the plate cross section (Fig. 29) should be limited to control the magnitude of the volume change or other structural deformation tensile forces which can be attracted to the connection. Generally, a 1/4 in. thick plate, 2 in. wide and 4 in. long (6.3 x 51 x 102 mm), can be used for A36 steel which limits the attachment plate's yield force to 18 kips (80 kN).

This type of controlled yield design is achieved with welds only at the plate ends to insure adequate deformation beyond initial yield. An alternate method of achieving the same result would be to use an angle welded only at its toes. The attachment plate of Fig. 29, when in tension due to volume changes (N_x forces), induces torsion into the spandrel that is additive to the torsion created by the gravity loads.

Erection connections can be part of

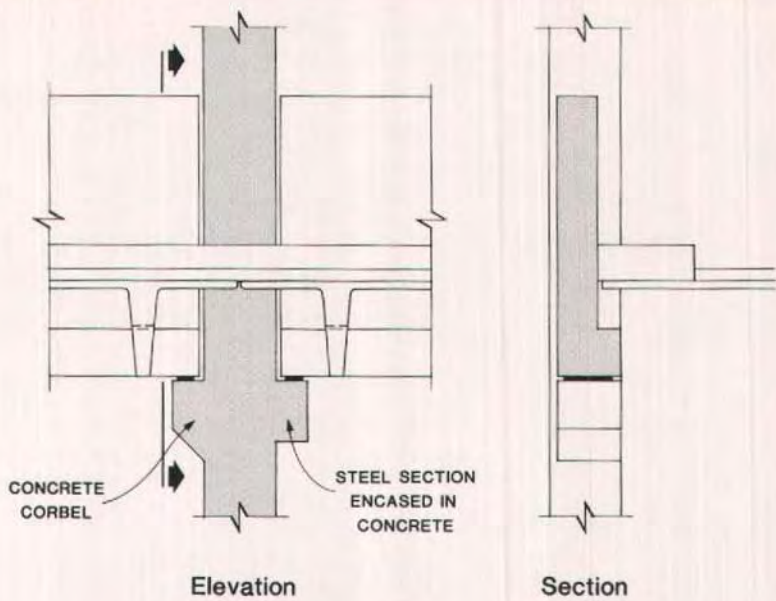


(a)

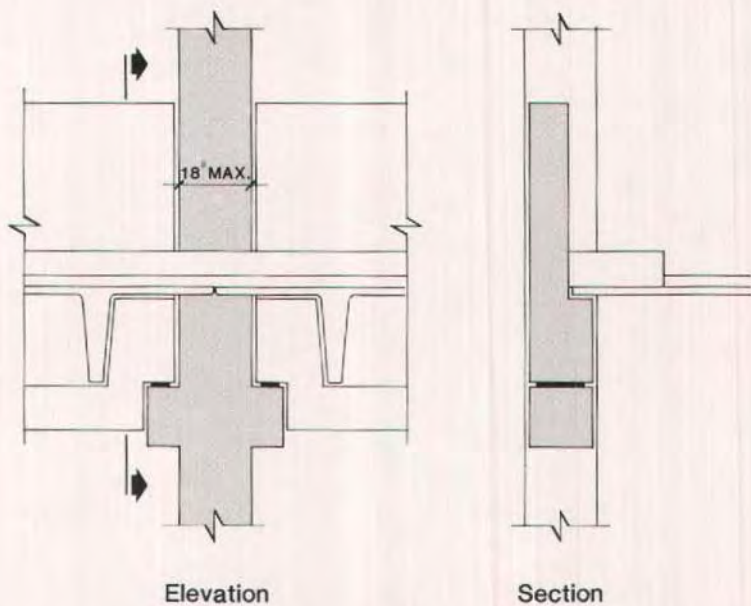


(b)

Fig. 27. Parking garage beam-column connections.

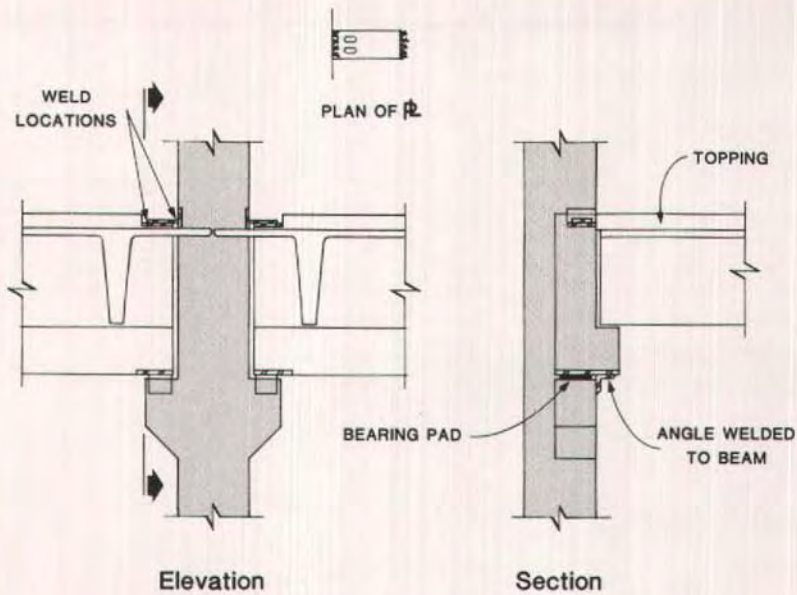


(c)

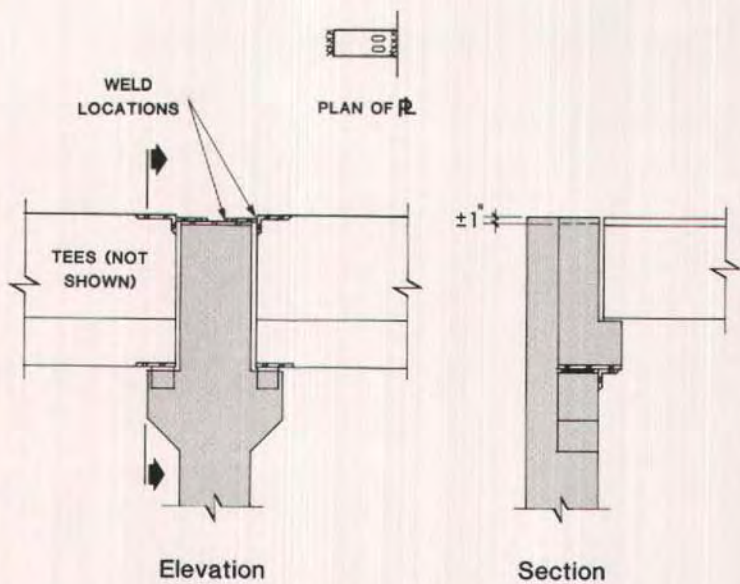


(d)

Fig. 27 (cont.). Parking garage beam-column connections.



(a) Floor Level



(b) Roof Level

Fig. 28. Building spandrels.

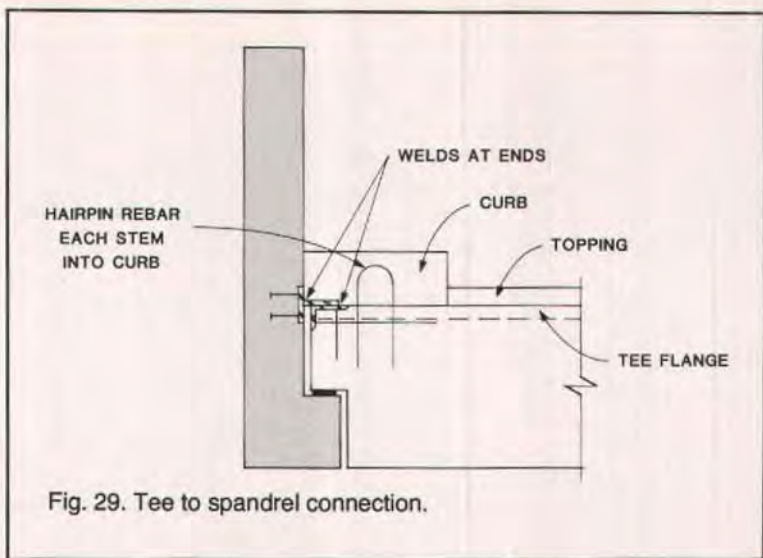


Fig. 29. Tee to spandrel connection.

the final permanent connection system, or they can be connections which are temporary and serve only during the erection process. The erection connection, if temporary, can remain in-place and be hidden within later construction (such as topping, curbs, and above ceilings), left exposed to view, or removed. Erection connections that are not removed also can serve as secondary connections tying the spandrel to the column.

Erection connections are needed to satisfy overall torsion equilibrium, for providing necessary structural ties between beams and columns and beams to tees (slab type members), and to develop the stability of the erection bracing. The stability requirements of the framing system during erection can induce forces into connections which are greater at the time of erection than the ultimate in-place loads to the completed frame. The design of erection connections for spandrels and other members generally requires a more comprehensive understanding of structural design and the criteria for the design than does just the design for final in-place conditions.

Connection Materials

A variety of readily available materials can be used to satisfy the forces and the behavior characteristics of spandrel beam connections. Among some of the common materials used in spandrel or other connection assemblies are bearing pads, concrete inserts, steel plates, welds, coil or threaded rods, reinforcing bars or stud type anchors, and concrete.

Bearing pads are used to transmit the spandrel reaction to the supporting column or to transfer loads to the spandrel ledge from the members supported by the beam. Pads likewise are employed in torsion equilibrium connections (see Fig. 27). The main functions of bearing pads are to provide uniform bearing by the pad's ability to accommodate non-parallel bearing interfacing surfaces and to minimize the development of volume change forces because of the pad's shear deformation characteristics. Most bearing pads (rubber, neoprene, cotton duck, and fabric reinforced rubber types) cannot be relied upon to develop torsional equilibrium couple forces if the couple force depends on the shear behavior or strength of the pad.

Concrete inserts can be used to satisfy many different connection functions. Inserts commonly are employed to provide the torsion equilibrium tension force couple as illustrated in Fig. 27, or to tie spandrel beams and columns to a horizontal diaphragm using threaded rods into concrete topping, curbs, or where concrete is the connecting medium. When selecting inserts for use in spandrels, the designer must consider whether the insert's capacity is controlled by the mechanical properties of the insert steel or the insert's concrete shear cone (full or partial).

Metal plates have a multitude of uses in spandrel beams. The plates can serve as a means to transfer or develop bearings, tension, compression, or shear forces. These plates can be made of A36 steel, higher strength steels, or stainless steels (generally AISI 304). Plates by definition can mean flat plates, bent plates, or angles.

The use of plates requires consideration of environmental factors relative to possible corrosion. Corrosion evaluations can result in the plate(s) not having any protective coating, having properly selected paints (primers and second coats) and surface preparations,⁷ being galvanized (per ASTM A153) with the necessary galvanized thickness, or being stainless steel. Plates used in parking garages, plates exposed to moisture, or plates exposed to salt water air (coastal regions), should be galvanized at a minimum. Plates used in other more aggressive conditions should be of stainless steel or be a suitable metal capable of resisting the specific chemical attack.

Welds are important in making spandrel beam connections. They are used to fabricate connection hardware, to complete connections between concrete embedded plates, and to attach reinforcing bars to plates. If welds are to serve their intended purpose, the type of the weld electrode, pre-heating requirements, environmental tempera-

tures at the time of welding, type of weld, weld heat distortion, weld shrinkage (cooling), and inspection (quality control) all must be considered.

Coil and threaded rods are selected to make many connection types. The coarse threaded coil rods find wide use in connecting spandrels to concrete curbs and topping, in addition to connecting columns supporting spandrels to horizontal diaphragms. Coil and threaded rods are used with concrete inserts requiring the insert capacity to be matched with the capacity of the rod. Coil rods, generally $\frac{1}{2}$ and $\frac{3}{4}$ in. (12.7 and 19 mm) in diameter, are most frequently selected for connections in cast-in-place concrete because of their coarse threads.

Reinforcing bars are the main part of a large number of spandrel connections such as ledges, webs, bearing, daps, and plate anchorages. Alternately, deformed bar anchors or headed concrete anchors can serve the same spandrel connection purposes. When using reinforcing bars for connections, attention must be paid to the influence of the bend radius (no rebar is bent at a right angle), weld requirements (bar chemical composition, low hydrogen electrodes, and pre-heating), the bar configuration, and placement tolerances.

Concrete is often selected for connecting spandrels to other structural members. Typically, cast-in-place concrete curbs and topping serve not only as the connection load transfer mechanism but also constitute the means to prevent axial volume change forces from being induced into the connections between spandrels and columns. The use of concrete as a connection device requires it to have the proper cross section, reinforcement, rebar cover, constructability, strength, durability, and mix quality.

Connection Interfacing

Similar to all precast concrete connections, spandrel beams require inter-

facing of the connection(s) with other parts of the beam and other structural members. Connection interfacing deals with tolerances, clearances, dimensional variations, skews, and position conflicts between connections and reinforcements within the beam.

Recommended tolerances for connections are given in the PCI Design Handbook.¹ But, when considering tolerances, evaluations must be made regarding the cumulative effect of all the tolerances involved. The critical tension insert, providing part of the spandrel overall torsion equilibrium of Figs. 27a and 27b, serves as an interfacing example.

The element which must be adjustable to accommodate the tolerances involved is the size of the insert hole in the beam (see Fig. 30). The tolerances shown in Fig. 30 that must be reviewed are:

- The as-fabricated location of the column insert as defined by dimensions a_c and b_c . The planned position of a_c and b_c can vary a possible maximum of $\frac{3}{8}$ to $\frac{1}{2}$ in. (9.5 to 12.7 mm), plus or minus.
- The a_s and b_s location dimensions of the hole in the as-made beam may vary a maximum of $\frac{3}{8}$ to $\frac{1}{2}$ in. (9.5 to 12.7 mm), plus or minus.
- Skew of the insert in both the x and y directions that cause the insert centerline to vary by as much as a $\frac{1}{4}$ in. (6.3 mm) at the beam face. Similarly, x and y skew of the hole in the beam can occur.
- The length of the spandrel beam c_s can vary up to $\frac{1}{2}$ in. (12.7 mm) or more causing the centerline of the beam hole to change relative to the column insert by the same amount.
- The location of the column can be different than planned because of the footing's location or the position of anchor bolts within the footing. The column erection position dimension c_c thus can vary by $\frac{1}{2}$ to 1 in. (12.7 to 25.4 mm).

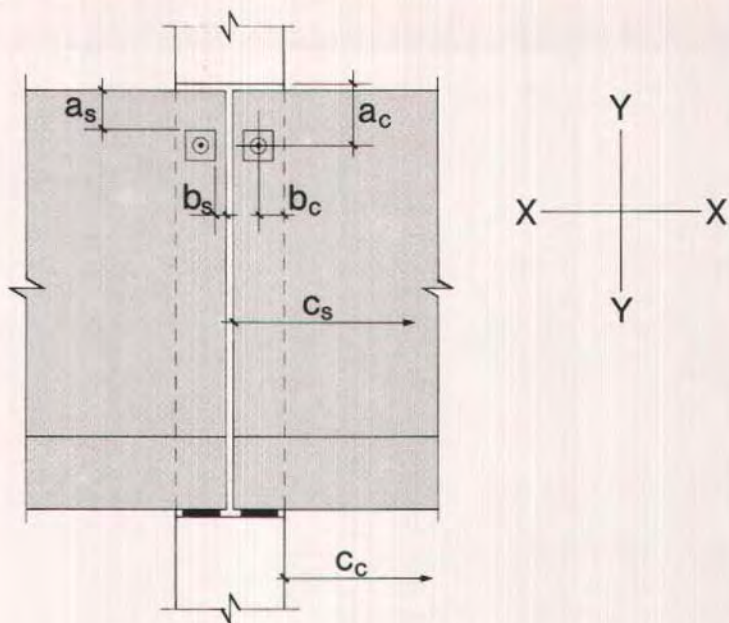
- The tolerance variations in dimensions a , b , and c can be in opposite directions and, combined with insert and beam hole centerline skew, thus result in a connection that cannot physically be made even though all tolerances are satisfied.

The foregoing pertains to any spandrel connection and connections in general (such as precast, steel, cast-in-place concrete, and masonry). Normally, tolerance variations of each dimension involved do not occur all at the same time, and some variations offset others. The previous tolerance example regarding spandrel equilibrium tension inserts does, however, demonstrate the need for an experienced engineer to review the design when selecting connections and their related clearances.

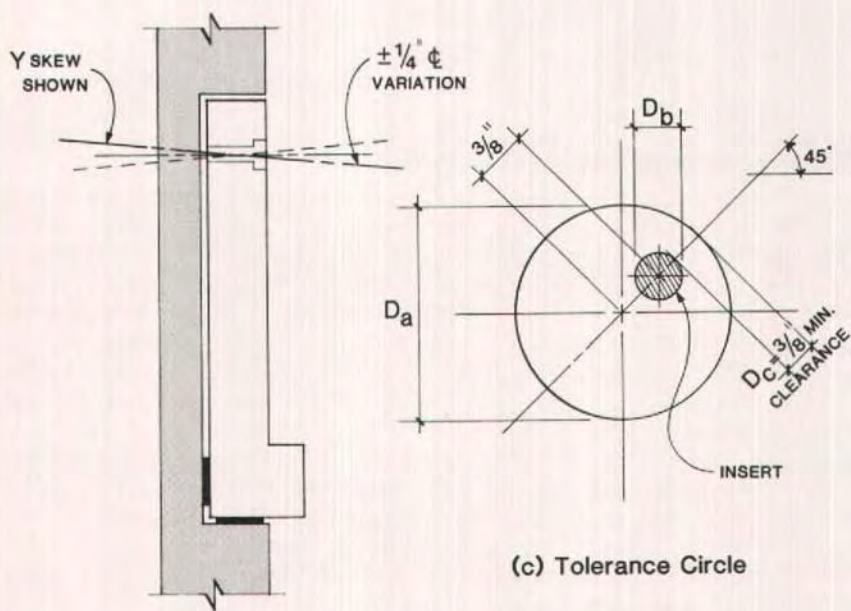
Interfacing tolerances are usually accounted for by selecting appropriate clearances. The insert tolerance circle of Fig. 30 provides an illustration of clearances. The hole diameter D_a can be selected once the insert rod diameter D_b is known, insert location tolerances are established, and the clearance dimension D_c between the rod and the hole side is determined. A D_a of $2\frac{1}{2}$ in. (63.5 mm) results for a 1 in. (25.4 mm) D_b , a $\frac{3}{8}$ in. (9.5 mm) insert tolerance, and clearance D_c of $\frac{3}{8}$ in. (9.5 mm). Allowing the beam hole to also have a location tolerance variation of $\frac{3}{8}$ in. (9.5 mm) results in D_a becoming $3\frac{1}{4}$ in. (82.6 mm).

Connection plates embedded into spandrel beams or other precast structural members encounter a variety of interfacing conditions. Bearing plate skew, positioning tolerances, and plate anchor interference represent the majority of interfacing concerns.

Skewed bearing plates, as shown in Fig. 31, must be considered when designing spandrel connections. The skew can result in highly nonuniform bearing pad stresses, alter significantly the location of applied loads to ledges or the connection supporting the spandrel, and



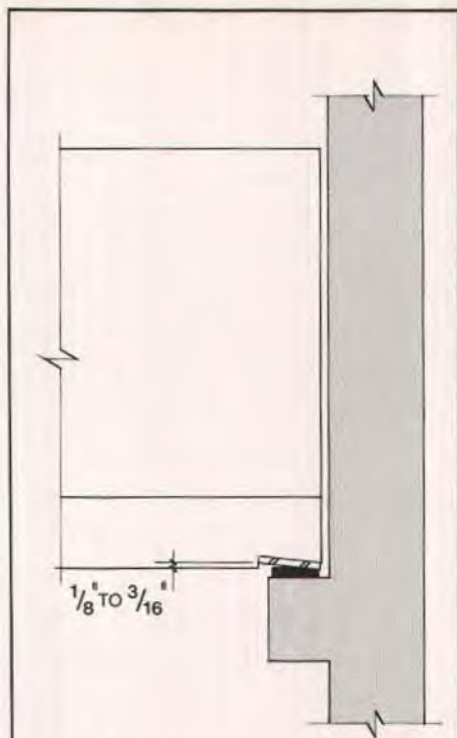
(a) Elevation



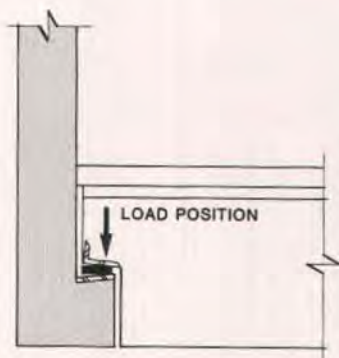
(b) Section

(c) Tolerance Circle

Fig. 30. Tolerance interfacing.



(a) Beam End Bearing Skew



(b) Ledge Bearing Skew

Fig. 31. Plate skew.

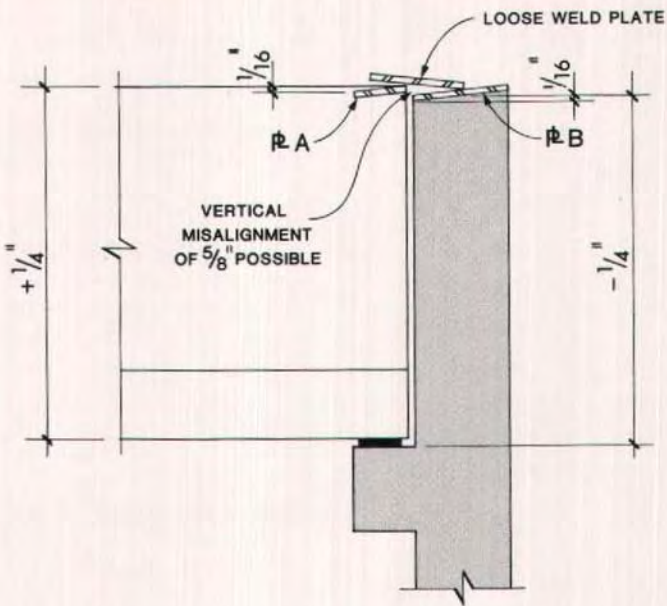
increase N_u forces on account of the plate skew cutting the bearing pad causing steel to concrete or steel edge loading with its increased friction coefficient. Concerning the bearing plate skew, a skew of $\frac{1}{8}$ to $\frac{3}{16}$ in. (3.2 to 4.8 mm) should be considered between *any* two corners of the plate.

Plate positioning and the related tolerances are influenced by plate anchor interference. The type, dimensional size, and the location of anchors should be evaluated at the time of design to determine if conflicts exist between any of the anchors and other materials or reinforcements within the beam. At a minimum, plate anchors require a clearance of $\frac{3}{4}$ in. (19 mm) from any other items in the adjacent concrete area unless rigid placement tolerances [$\pm \frac{1}{8}$ in. (3.2 mm)] are absolutely assured.

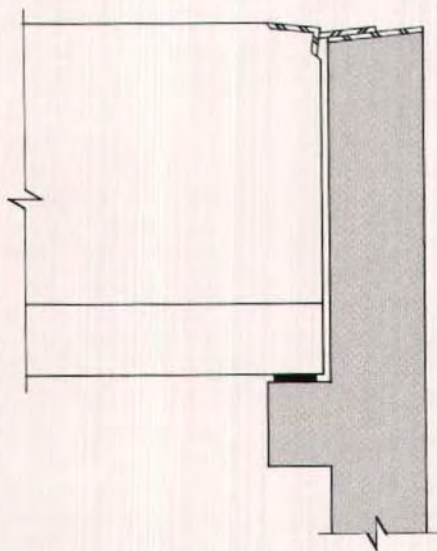
When plates are positioned in beams, and no anchor interference develops, a plate position tolerance in plan or elevation of $\pm \frac{1}{2}$ in. (12.7 mm) in any direction can be expected to occur. Position tolerances for plates apply to plates on both sides of the connection. It is not unusual to observe one plate being out of position by $+\frac{1}{2}$ in. (12.7 mm) and the other by $-\frac{1}{2}$ in. (12.7 mm), creating a combined misalignment of 1 in. (25.4 mm). Positioning tolerances, like all other practical real world tolerances, are cumulative.

An example of cumulative tolerances compromising the effectiveness of a spandrel connection is shown in Fig. 32a where it is incorrectly assumed that both beam and column plates will be flat, level, and aligned. Details similar to Fig. 32b should be used since they are not tolerance sensitive. Proper planning and consideration of tolerance can eliminate most tolerance interfacing problems.

Connection interfacing difficulties can develop independent of tolerance variations. Fig. 33 presents the bearing pad interfacing between the spandrel ledge and a long span prestressed dou-



(a) Accumulative Tolerances



(b) Detail Providing Interfacing

Fig. 32. Tolerance interfacing.

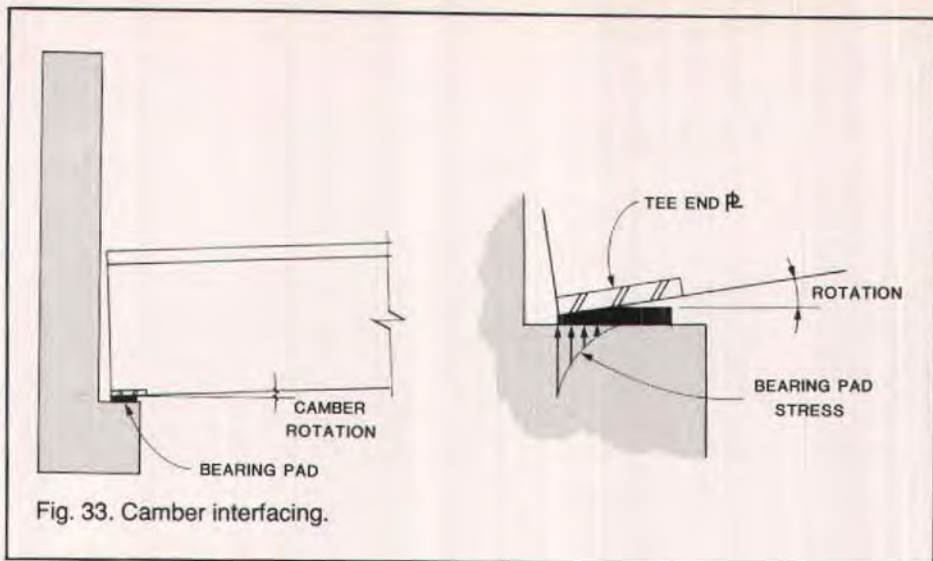


Fig. 33. Camber interfacing.

ble tee's end camber rotation. The camber rotation results in nonuniform pad bearing, and depending upon the magnitude of the nonuniform stress can damage the bearing pad. Plates cast in the tee end can be skewed thereby further increasing the nonuniform bearing stress or exaggerating the camber effect. Camber interfacing requires the design anticipating nonuniform bearing and selecting the proper type of bearing pad.

A variation of the plate interfacing tolerances is shown in Fig. 34 where shared bearing develops. Shared bearing consists of the bearing pad contacting both the concrete and steel in the bearing area. The use of shared bearing can be satisfactory only when interfacing is perfect. Commonly, shared bearing creates distortions and distress in bearing pads, thereby reducing their effectiveness or causing damage. Conditions which can result in shared bearing should always be avoided. Fig. 34a relates to shared bearing difficulties while Fig. 34b shows the method to prevent it. The common bearing of Fig. 34b employs a plate in the beam's end larger than the one in the column corbel.

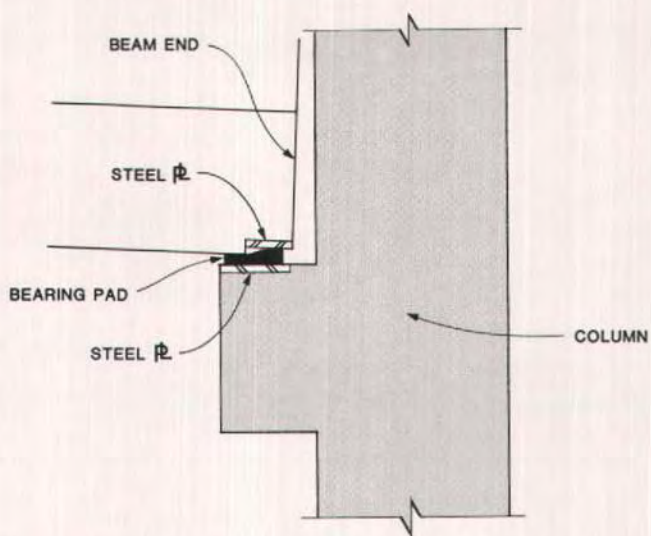
Interference of plate anchors with

other materials within the beam is only one type of connection component interfacing. Spandrel beam connections employ a range of materials as previously discussed. All of these materials, such as inserts and reinforcements, must fit together. Interfacing fit-up problems in most instances are the direct result of inadequate design planning and the preparation of improper schematic type details.

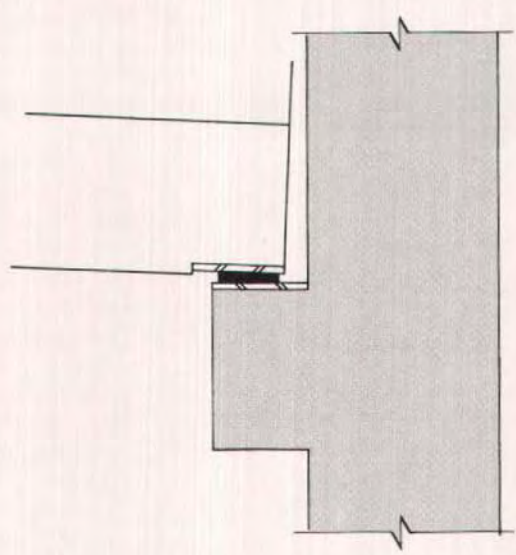
Proper connection internal fit-up interfacing requires the designer to prepare details showing all materials which exist at the connection location. Further, the details must illustrate the items occurring in the plane of the detail in addition to other items perpendicular to the detail plane. Connection fit-up interfacing requires three-dimensional evaluations of all materials of the connection. ACI 315-74,⁸ Chapter 6, regarding design guidelines provides an excellent summary of interfacing.

Frame Connections

Frame connections between spandrel beams and columns can be achieved by post-tensioning, reinforcement and



(a) Shared Bearing



(b) Common Bearing

Fig. 34. Shared support.

cast-in-place concrete, or welding. The usual method is by welding, and hence only welding will be discussed.

A basic criterion of precast frame connections is to use only those needed and no more. This criterion is based upon the greater costs associated with frame connections and the volume change forces attracted to the connections because of increased column and beam fixity stiffness. Two types of full frame beam-to-column moment connections are given in Fig. 35. Both types can provide the same moment capacity.

The flat plate type (see Fig. 35a) has several disadvantages as discussed below. Welding destroys whatever corrosion protective coating has been applied, thus requiring field touch-up which does not have the same protective ability as the original coating. Also, the flat plate connection results in hard bearing creating the possible need of some internal bearing reinforcement to provide for bearing stress concentrations. The top connection pocket shown in Fig. 35a requires a filler, and based on actual experience the filler needs periodic maintenance.

The vertical plate connection (Fig. 35b) employs a 4 x 6 in. or 4 x 8 in. (102 x 152 mm or 102 x 203 mm) grout column to provide for shear transfer and also give corrosion protection to the weld plates. The grout shear keys minimize reaction shear to the vertical connection plates which can result from frame action or elastic and creep deformations of the thin [$\frac{1}{4}$ to $\frac{3}{8}$ in. (6.3 to 9.5 mm)] bearing pad. A drawback to the vertical plate connection is the dimensional variation in the 4-in. (102 mm) gap caused by member and erection tolerances. Back-up bars used with full penetration welds allow for a dimensional variation of $\frac{1}{4}$ in. (6.3 mm). Different sized plates are required to accommodate the tolerance variations [viz., 3 $\frac{1}{2}$, 4, and 4 $\frac{1}{2}$ -in. (89, 102, and 114 mm) plate widths].

The details of the embedded plates

constituting the frame connections (Fig. 35) are illustrated in Fig. 36. The angle plate (Fig. 36a) requires not only reinforcing bars to resist the H force, but also additional bars to counter the moment created by the eccentricity of the H forces. The vertical or horizontal plate (Fig. 36b) welded to the embedment should employ a structural steel tee or a flat plate with stiffeners. The tee stem or stiffeners are necessary to transfer the loose plate force directly to the reinforcing bars of the embedment. Moreover, the distribution of the loose plate force may not be even to the embedded reinforcing bars considering tolerance variations such as dimensional offsets in the placement of the connection hardware in the beam and column.

The use of welded frame connections emphasizes the need to consider the sequence of the beam-to-column welded connections and at what stage of the construction these connections will be made. Typically, if the beam is subjected to gravity loads, the frame lateral stability connections should not be made until all or most of the dead load is in place. The sequence of making the beam-to-column frame welded connections should be that which minimizes the frame's volume change forces resulting from weld shrinkage.

Structures which use spandrels or other beams in a moment frame to resist lateral loads and to provide the necessary lateral stability require their welded frame connections to be properly inspected. The weld inspection and related testing must be applied to the parts embedded in the beams and columns in addition to the field welds. If rigorous welding inspection and testing are not employed, welded frame connections should not be used. Likewise, proper specifications are required to determine the welding requirements and procedures as controlled by the reinforcing bar chemical composition and weld electrodes employed.

Frame analysis to determine forces in

EPOXY FILL OR
EPOXY GROUT
BLOCKOUT

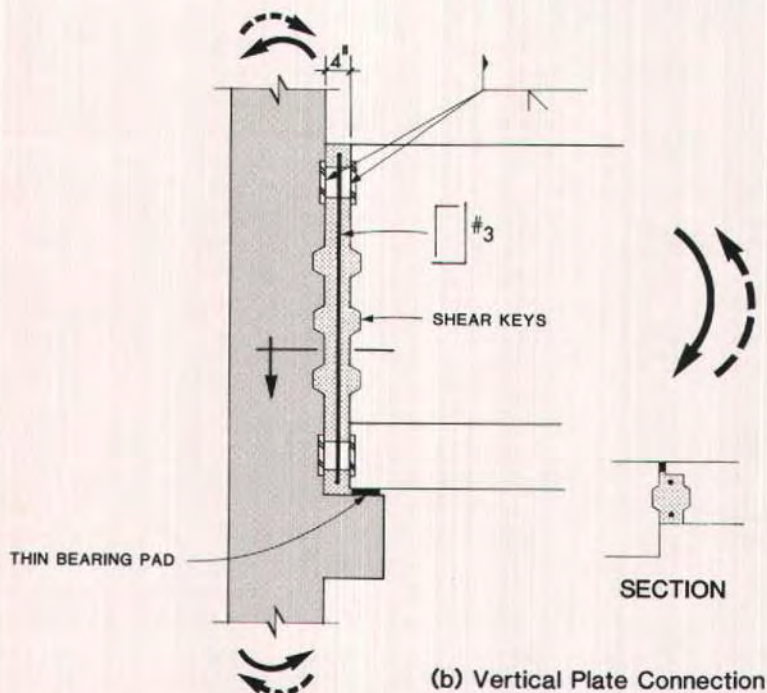
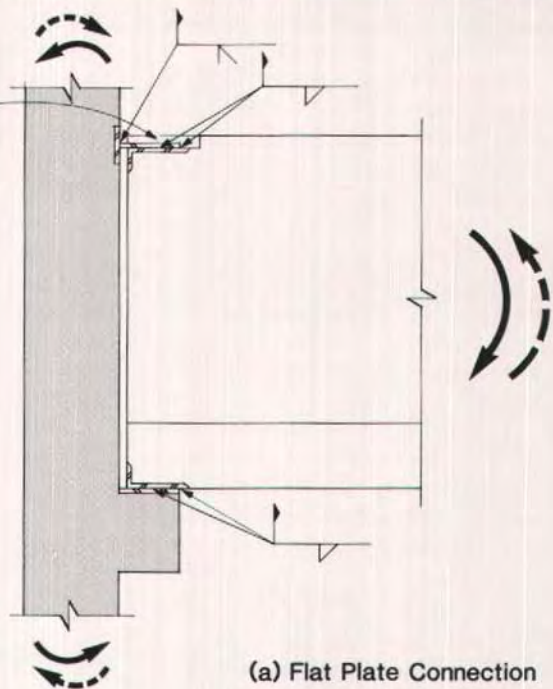
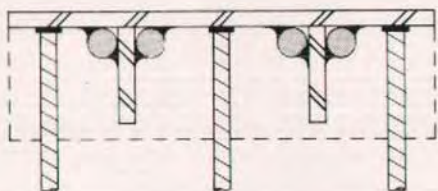
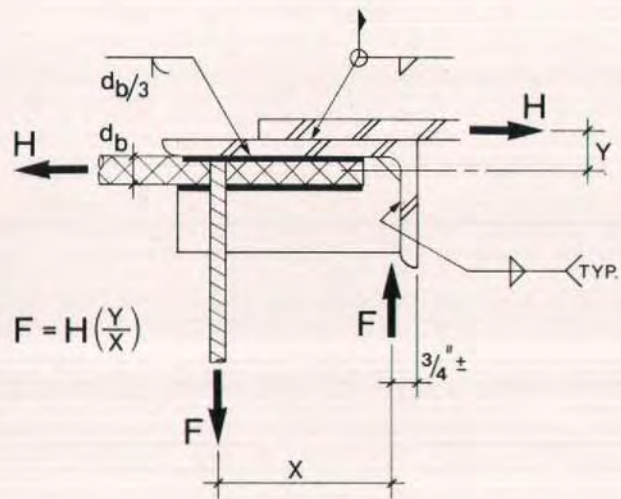
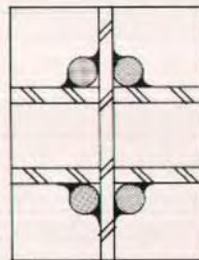
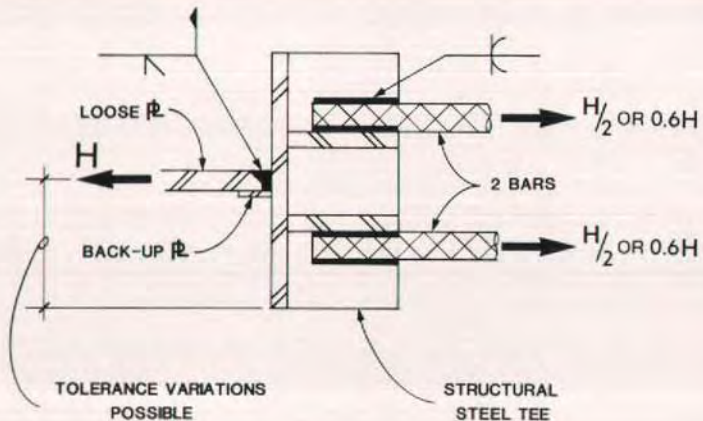


Fig. 35. Welded frame connections.



(a) Angle Plate



(b) Vertical or Horizontal Plate

Fig. 36. Frame weld plate details.

the beams and columns, and to calculate the frame deformations, must consider the effect of the various member sizes. This is necessary if the proper stiffness of the column is to be correctly evaluated. Figs. 37a and 37b reflect the computer analysis models for the Fig. 35 frame connections. The more convenient analysis model of Fig. 37c cannot be used since it is incorrect and will produce erroneous results. For example, if the floor-to-floor dimension of the column is 10 ft (3.05 m), the column length for Fig. 37c would be 10 ft (3.05 m) whereas the clear column height of Figs. 37a and 37b could be only 6.5 ft (2 m). A difference in the column height of 3.5 ft (1.1 m) has a significant influence on the magnitude of the volume change forces induced into the frame connections, and upon the beam and column forces.

Reinforcement Considerations

Spandrel beam ledge and end support connections use internal reinforcement in connections. The reinforcement across critical crack planes and for bearings must have proper anchorage and be correctly oriented if it is to perform its function. Additionally, the connection steel requires more accurate placement than the usual beam flexural and shear reinforcement. And, if the spandrel is used in an exterior environment, concrete cover and reinforcement corrosion demand increased attention.

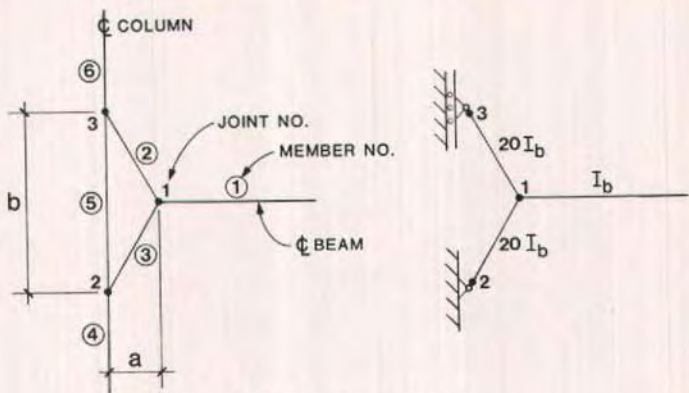
Ledge reinforcement, as shown in Fig. 38, can serve solely as flexural steel or as combined flexural and confined bearing steel. The reinforcement of Fig. 38a should be considered only in providing for ledge flexure, and the bearing design for V_u and N_u should employ plain concrete analysis methods since the critical bearing failure crack plane can by-pass the reinforcing steel because of its end bend radius. If the ledge reinforcement is also to accommodate bearing (ultimate bearing stresses exceed that allowed by plain concrete bearing),

proper anchorage must be insured as shown in Fig. 38b. Anchorage for bearing can be achieved either by welded cross-bars or alternate methods using reinforcing bars bent and placed in a manner that will provide the positive anchorage. Often, supplemental bearing reinforcement is supplied using plates and deformed bar anchors, as given in Fig. 39, to develop the required confined bearing capacity.

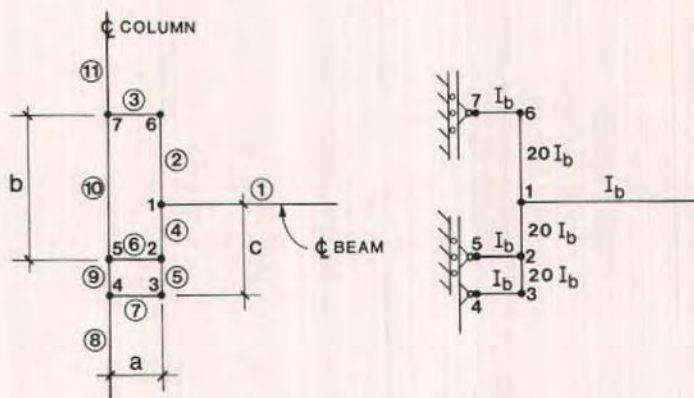
The location of the ledge connection reinforcement and all other spandrel connection reinforcing bars is important to the connection in developing its design strength. Therefore, specified dimensions for positioning the reinforcement are essential. Along with providing the steel's location, specific positioning tolerances must be determined. A realistic tolerance for controlling critical rebar positions should be $\pm 1/4$ in. (6.3 mm) as shown in Fig. 38.

The concrete cover dimension c for precast spandrel beams produced under plant controlled conditions and exposed to weather should not be less than $1\frac{1}{4}$ in. (31.8 mm)³ for #5 bars and smaller. However, if the spandrel is exposed to a severe corrosive environment (chloride salts) the minimum cover should be $2\frac{1}{4}$ in. (57.1 mm) for connection rebars if a $1/4$ in. (6.3 mm) tolerance is specified. Also, severe exposure may require the spandrel's concrete to have a maximum water-cement ratio of 0.4 by weight⁹ to provide the necessary corrosion protection for steel.

The design planning of reinforcement for connections and other reinforcing bars in spandrel beams requires equal or greater skill than the reinforcement based on structural design. The spandrel's reinforcement design, selection of bar sizes, bend radius, concrete cover requirements, and interfacing of all the beam's steel can control or dictate the spandrel beam's cross-sectional dimensions as can the width of the compression flange relative to the beam's span (width = span/50).



(a) Fig. 35a Model

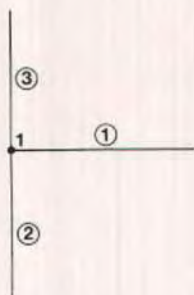


(b) Fig. 35b Model

a = DISTANCE ϕ COLUMN
TO BEAM REACTION

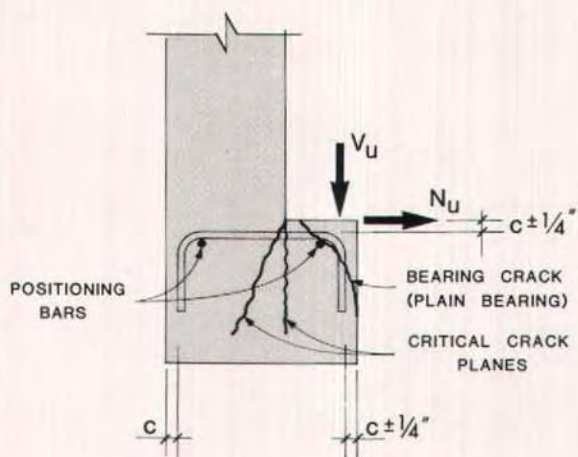
b = DISTANCE BETWEEN
CONNECTIONS

C = DISTANCE ϕ BEAM
TO BEAM BOTTOM

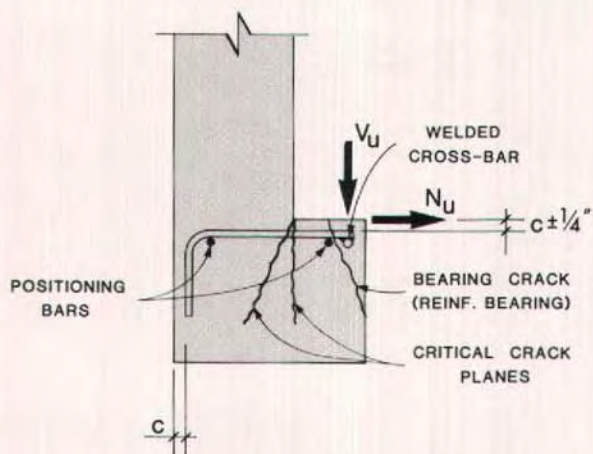


(c) Centerline Model

Fig. 37. Computer analysis models.



(a) Ledge Flexure Only



Alternate Ledge
Rebar Details

(b) Ledge Flexure & Bearing

Fig. 38. Ledge reinforcement.

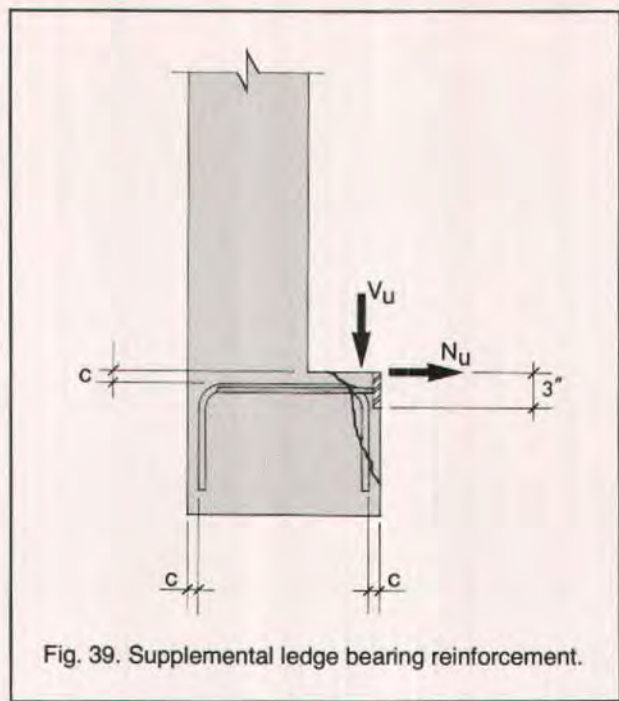


Fig. 39. Supplemental ledge bearing reinforcement.

Fig. 40 shows a typical arrangement of reinforcement. It becomes immediately apparent that concrete cover and bending radius limit the locations of other steels. The 2-in. (51 mm) grid for locating the prestressing strands is not completely available. Study of reinforcing bar bends can be important because the bar bends influence where other reinforcing bars can be positioned. Another reinforcing bar consideration relates to some bars being necessary to secure or provide for dimensional positioning of key steel, such as the ledge A_s bars.

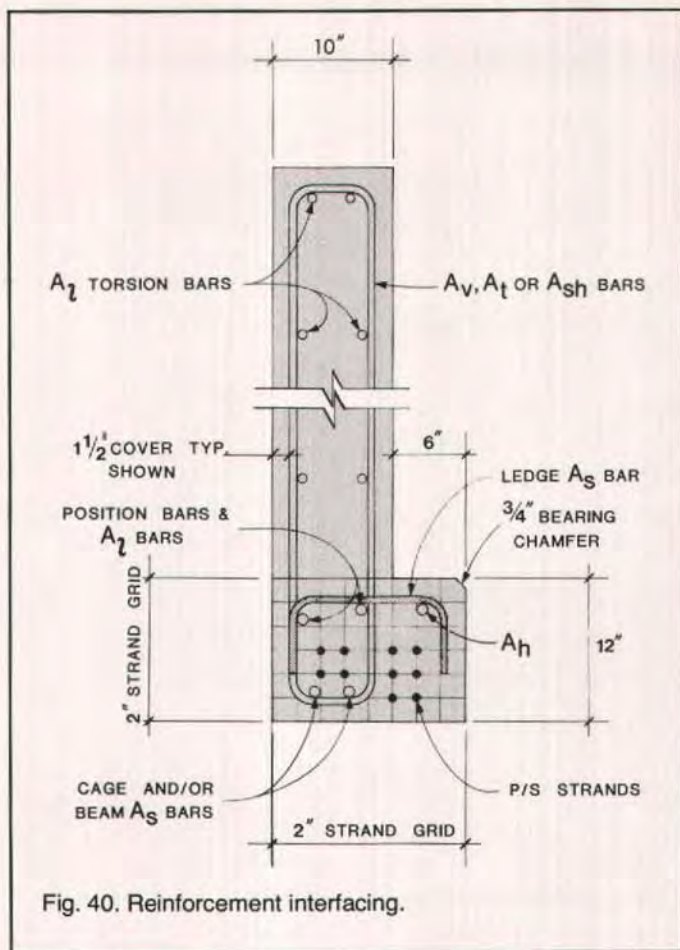
An additional factor to be evaluated regarding the influence of the reinforcement dictating, in part, the positioning of the prestressing strands (see Fig. 40) is the non-alignment of the strand's centroid with respect to the spandrel's principle Y_p axis (see Fig. 26). This non-alignment can result in horizontal sweep of the beam.

Fig. 40 shows reinforcement arrangements without any provisions for

tolerance. Tolerances for locating the various bars within the cage and within the beam must be considered, as must the bars' having an outside diameter of $\frac{1}{8}$ in. (3.2 mm) greater than their normal size on account of the ribs. Fabrication tolerances for individual bent bars must also be evaluated. The fabrication tolerance for stirrups and ledge A_s bars is $\frac{1}{2}$ in. (12.7 mm) on specified dimensions plus another $\frac{1}{2}$ in. (12.7 mm) for out of square configuration.

The preceding discussion on spandrel reinforcements, and the many criteria the bars must satisfy, emphasizes the fact that the designer must select the three-dimensional reinforcements and make sure his design can actually be built. A cause of many spandrel beam problems and distresses is that beam reinforcements are selected without any concern for interfacing between them, and critical reinforcements end up significantly out of position.

The reinforcement considerations



presented herein relate to ledge and web reinforcement alone. The reinforcement concepts discussed apply equally to spandrel beam ends with regard to daps, ledge corbel behavior, or beam end torsion. Typical connection reinforcement details can only be typical when they interface with *all* reinforcement conditions.

Column Influences

Connections which attach the spandrel beam to the column for the basic purpose of supplying the beam's overall torsion equilibrium can be subjected to additional column equilibrium loads.

The influence of column equilibrium loads on the spandrel torsion stability connections is even more pronounced if no direct connection is made between the column and the horizontal diaphragm (floor or roof level). In this instance, additional horizontal forces are induced into the beam end torsion equilibrium connections which can represent a marked increase in the required connection design load. Spandrel connection designs which do not consider the additional column equilibrium forces have resulted in connection failures and expensive connection repairs.

Depending upon the framing ar-

angement of the spandrels to the column, the analyses to determine beam end connection forces can be very complex. Fig. 41a illustrates the framing and pertinent dimensions for a two-story parking garage where the spandrel beam torsional equilibrium is provided by end connections. In addition to the end connections, the column is attached directly to the horizontal diaphragms as is the beam for its lateral support. The diaphragm horizontal connections are achieved with inserts, coil rods, and concrete topping. The horizontal diaphragm is in turn laterally supported by shear walls or moment frames.

The analysis to determine the connection forces resulting from both beam torsion equilibrium and column equilibrium is best made using computer methods. The analytical model and requirements of the model to duplicate the actual frame behavior are presented in Fig. 41b. The spandrel beam vertical span properties (see Fig. 41c) can be determined from the web width of 8 in. (203 mm) acting over the lesser horizontal length of eight times the web width or the total height of the spandrel beam.

The model (Fig. 41b) deals only with the additional column equilibrium forces to which the beam-to-column connections are subjected. Another loading condition which must be reviewed is that resulting from volume change deformations acting parallel to the tee span. The beam-to-column connections can be considered as not receiving any additional loadings from volume change movements providing a direct column-to-diaphragm connection exists. However, when no column-to-diaphragm connection is used, the additional connection forces must be accounted for. Instead of the rigid horizontal supports of Fig. 41b, horizontal framing members, per Fig. 42, should be used to simulate the volume change deformations so the total forces to the connections can be determined.

GOOD DESIGN PRACTICE

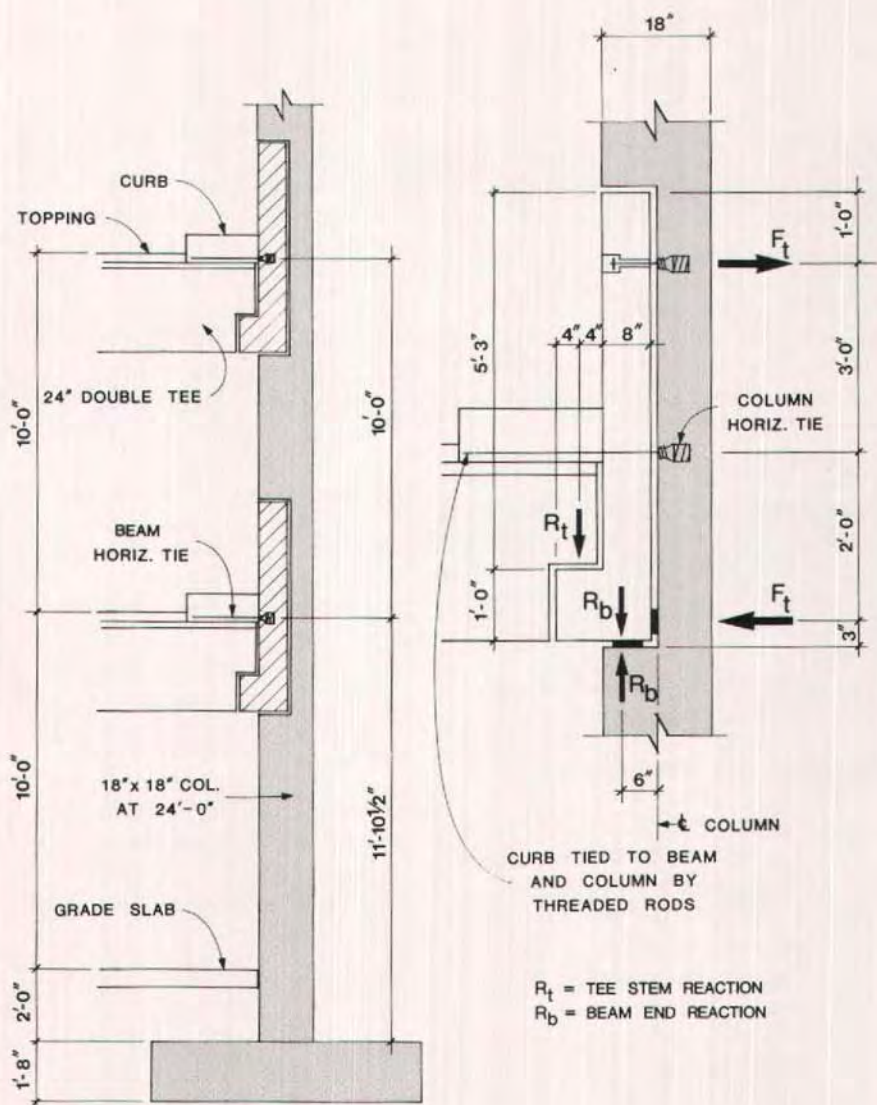
The design of spandrel beams encompasses not only an understanding of engineering fundamentals but also properly requires an appreciation of practicality and experience. The following outline on general design guidelines for spandrels is based in large part upon observations of past performance, both good and bad. Among the many aspects involved in spandrel beam design, those set forth represent some of the more pertinent factors.

Cross-Sectional Dimensions

- The spandrel web width generally should not be less than 10 in. (254 mm), and never less than 8 in. (203 mm).
- The horizontal ledge projection should not be less than 6 in. (152 mm).
- The vertical height of the ledge should not be less than 10 in. (254 mm), and for beams supporting large concentrated reactions a 14-in. (356 mm) height or greater is realistic.
- The final beam cross-sectional dimensions may be dictated by the needs of practical non-congested reinforcement arrangements to insure the spandrel can be realistically built within its selected dimensions.
- Blockout and connection fit-up requirements in spandrels, particularly moment frame beams, may control the cross-sectional dimensions if all internal components are to be properly integrated within the beam's final dimensions.

Reinforcement

- Bars have a radius at bends, and are not bent at a sharp 90-deg angle.
- Fabricated reinforcing bar configurations have tolerances and usually will be out of square.



(a) Framing Arrangement

Fig. 41. Connection loads induced by column equilibrium.

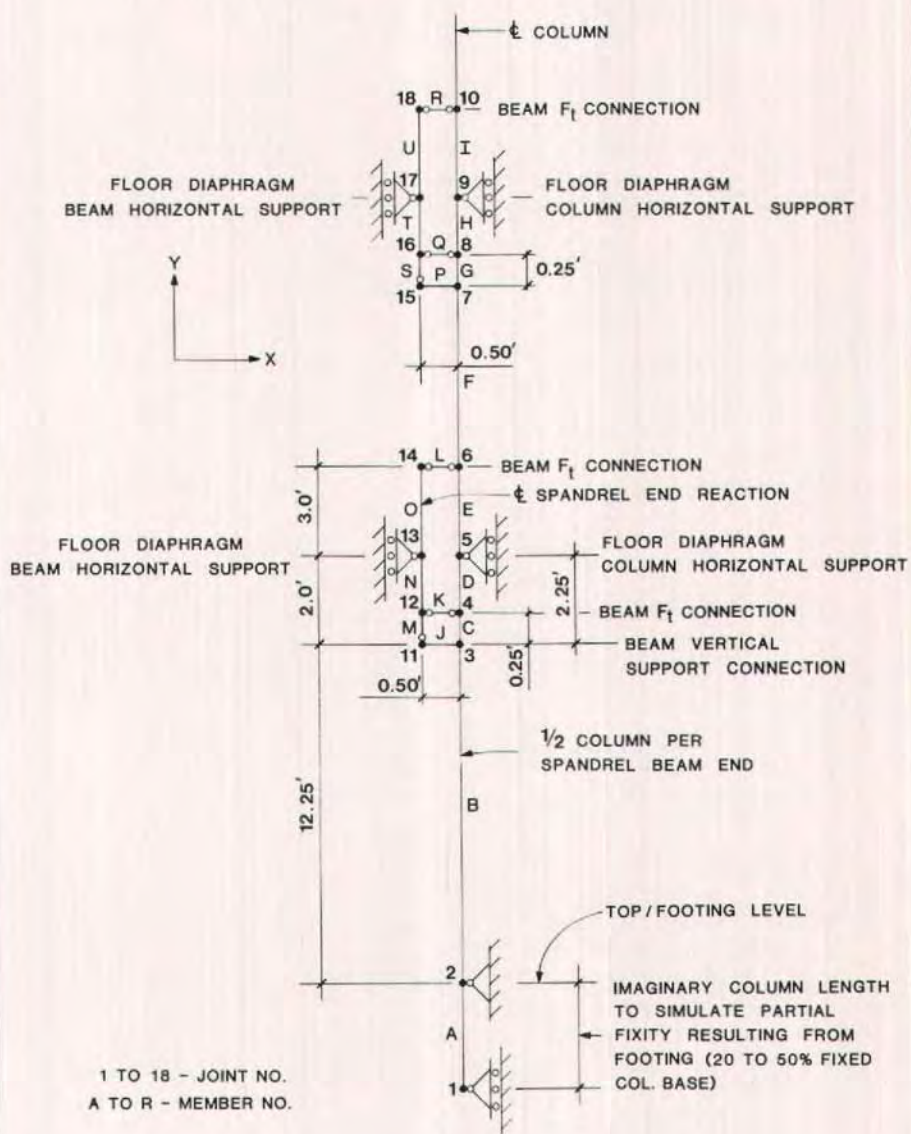


Fig. 41 (cont.). Model for analyzing connection loads induced by column equilibrium.

SUPPORT JOINTS

<u>Jt. No.</u>	<u>Type</u>
1	pin - y roller
2	pin - no rollers
5	pin - y roller
9	pin - y roller
13	pin - y roller
17	pin - y roller

JOINT LOADS
(Service Loads)

<u>Jt. No.</u>	<u>Loads</u>
11	$\Sigma P = 56K$ and Couple* = 28 Ft.-K
15	$\Sigma P = 56K$ and Couple* = 28 Ft.-K

* Couple acts counter-clockwise

MEMBER PROPERTIES

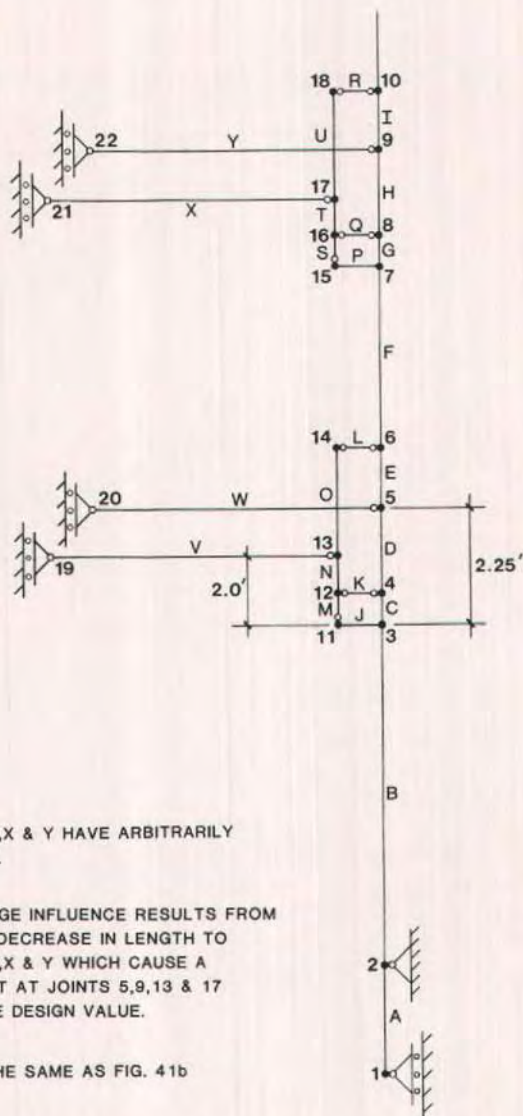
<u>Member</u>	<u>End Jts.</u>	<u>I - in⁴</u>	<u>A - in²</u>	<u>Joint Releases</u>
A	1,2	8748/2	324/2	none
B	2,3	8748/2	324/2	none
C	3,4	8748/2	324/2	none
D	4,5	8748/2	324/2	none
E	5,6	8748/2	324/2	none
F	6,7	8748/2	324/2	none
G	7,8	8748/2	324/2	none
H	8,9	8748/2	324/2	none
I	9,10	8748/2	324/2	none
J	3,11	9000*	0.1**	none
K	4,12	0.1**	9000*	pin @ each end
L	6,14	0.1**	9000*	pin @ each end
M	11,12	2731	512	pin @ 11
N	12,13	2731	512	none
O	13,14	2731	512	none
P	7,15	9000*	0.1**	none
Q	8,16	0.1**	9000*	pin @ each end
R	10,18	0.1**	9000*	pin @ each end
S	15,16	2731	512	pin @ 16
T	16,17	2731	512	none
U	17,18	2731	512	none

* Arbitrarily selected small to simulate an x-roller or to simulate no flexural stiffness

** Arbitrarily selected large to simulate axially rigid connection or to simulate flexural rigidity

(c) Analysis Data

Fig. 41 (cont.). Data used in computer model for analyzing connection loads induced by column equilibrium.



NOTE:

1. MEMBERS V,W,X & Y HAVE ARBITRARILY LARGE AREAS.
2. VOLUME CHANGE INFLUENCE RESULTS FROM ASSIGNING A DECREASE IN LENGTH TO MEMBERS V,W,X & Y WHICH CAUSE A DISPLACEMENT AT JOINTS 5,9,13 & 17 EQUAL TO THE DESIGN VALUE.
3. REMAINDER THE SAME AS FIG. 41b

Fig. 42. Computer model for analyzing connection loads induced by column equilibrium and volume change deformations.

- Actual bar diameters, considering ribs, are about $\frac{1}{8}$ in. (3.2 mm) greater than their nominal size.
- Spandrel internal reinforcement for connections typically should be #3, #4, or #5 bars.
- Details should be prepared during design to check that the three-dimensional interfacing requirements for reinforcing bars are satisfied.
- The arrangements and positions of the steel must be such that ordinary workmen can fabricate and construct them.
- Clearances of $\frac{3}{4}$ in. (19 mm) between spandrel reinforcements in different planes should be planned to accommodate reinforcing bar tolerances and interfacing.
- Special welding requirements and procedures should be determined whenever welded reinforcing bars are used at critical connections.
- Reinforcing bars should not be welded near cold bends, and should *never* be tack-welded.

Tolerances and Clearances

- A 1-in. (25.4 mm) minimum clearance between spandrel beams and all other framing members should be selected. Long span spandrels, spandrels of large depth [48 in. (1.22 m) and greater], and spandrels supporting members spanning 60 ft (18.3 m) or more may require clearances between the other structural components of up to $1\frac{1}{2}$ in. (38 mm) to provide for tolerance fit-up interfacing.
- Separate evaluations of fabrication and erection tolerances should be made for each design rather than relying solely on "usual" industry tolerances.
- The influence of structural deformations during erection should be considered when selecting tolerance and clearance requirements.

- Tolerances generally accumulate and this effect should be considered.
- Designs should be based upon the tolerances and clearances selected which produce the greatest forces.

Corrosion Protection

- Concrete cover to reinforcing bars, considering placement tolerances, should be selected considering all influencing factors.
- Coatings to metal plates should be specified in detail and not be referenced as simply rust inhibitive paint or galvanized.
- Welding to hardware in beams that have corrosion protective coatings should be done prior to the coating application.
- Parking garages using metal plates exposed to the elements should require all plates to be galvanized at a minimum.

Loads

- Loads to spandrel beams and their connections during erection may be greater than the permanent design loads.
- The design should consider the combined effect of the different possible loading cases.
- The influence of volume change loads axially and rotationally, and other deformation caused loads, should be accounted for in the design. All connections should be designed for a minimum N_u tension force equal to $0.15V_u$.
- Loads to spandrels generated by the beam's torsional equilibrium requirements and by the supporting column's lateral equilibrium forces should be determined.

Connections

- Connections or other means must be used to provide for spandrel

beam overall torsion equilibrium at the time of erection and for the in-place permanent conditions.

- Where possible, avoid using tension insert connections to provide for spandrel beam torsion equilibrium because of tolerances.
- Consider all load combinations from construction to final in-place forces when designing and detailing spandrel connections.
- Evaluate the influence of tolerances on all the various spandrel connections.
- Do not count on bearing pad shear behavior as a means of providing overall beam torsion equilibrium.
- Select connection details which are not tolerance sensitive.
- Insure that connection hardware can be practically integrated into the spandrel without interference or conflict with other materials within the beam.
- Connections using loose field welded plates or threaded bars should require the anchoring hardware embedded in the beam and/or the adjoining member to have a load capacity one-third greater than the loose connecting plate.
- Complete free-body diagrams should be developed for each connection design to insure that all forces are accounted for and that statics are satisfied.

Bearing Considerations

- Employ plain concrete bearing in combination with bearing pads whenever possible.
- At service load levels, nominal P/A bearing stresses for ledges and beam ends should be in the 800 to 1000-psi (5.5 to 6.9 MPa) range.
- Avoid using plates for bearing surfaces whenever practically possible.
- When using plain concrete bearing, always provide some mini-

mum internal reinforcement to resist N_u forces.

- Consider using preformed pads composed of synthetic fibers and a rubber body, $\frac{3}{8}$ to $\frac{1}{2}$ in. (9.5 to 12.7 mm) thick, for most bearing conditions.
- Avoid using neoprene pads for general bearing conditions except when control of volume change deformations (not at expansion joints) is a critical design parameter, and then use only non-commercial structural neoprene grades satisfying AASHTO specifications.

Ultimate Load Factors

- Loads resulting from volume change deformations should employ the same ACI Code 1.7 load factor as required for live loads.
- All spandrel beam connections should use a load factor of at least 2 ($2 = 4/3$ the usual combined ultimate dead and live load factor of 1.5).
- Connections which rely upon concrete shear cones for capacity should use a load factor of 3 unless a lower load factor of 2.5 can be prudently justified.

Supporting Columns

- Loads to the column resulting from spandrel beam torsion equilibrium connections and the beam reaction should be analyzed to insure that the column is correctly designed.
- Column design requires that the influence of the column's and the spandrel's actual size be considered by the analysis rather than just using simple beam-to-column centerline dimensions for the analysis.
- Spandrel beams at the uppermost level, or elsewhere, should not bear directly upon column tops but instead be supported upon column

haunches or corbels, thereby providing for the top of the column to be approximately level with the top of the beam.

Inspections

- Connection reinforcements should be inspected for placement position and arrangement by the designer or his representative during spandrel beam fabrication.
- Welded moment frame connection hardware embedded in spandrels (and columns) requires weld inspection and testing of the rebar and plate welds.
- Field welds employed to complete moment frame connections require weld inspection and testing.

CLOSING COMMENTS

The behavior, design considerations, design relationships, and general guidelines on spandrel beams as presented are based upon the author's design, construction, failure investigation, and repair experience.

Spandrel beam design, if it is to be successful, requires consideration of the following items:

- No magic details or solutions exist. Each project's design requires the engineer to evaluate all the circumstances, determine the design criteria, and develop the appropriate solutions.
- Spandrel beam design cannot be

considered as only a simple component design. Rather, there must be "one" in-charge designer to insure that the spandrel beam's design satisfies the overall criteria, that building frame interfacing structural requirements are met, that deformation and restraint conditions have been evaluated, selected tolerances and clearances are satisfactory, and the connections have been correctly integrated.

- The hindsight key to good spandrel beam design is that the in-charge designer be meticulous regarding requirements and details.

The concepts discussed and the requirements reviewed for spandrel beam design apply equally to all aspects of precast prestressed concrete design, and to engineering design in general.

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Note: Discussion of this paper is invited. Please submit your discussion to PCI Headquarters by November 1, 1984.

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Note: An Appendix giving design examples is included following the Notation Section.

NOTATION

- a = distance from end of beam to horizontally applied load or height of ultimate strength rectangular stress block
 A_h = longitudinal reinforcement in beam ledge near ledge top, sq in.
 A_t = longitudinal torsion reinforcement about beam perimeter, sq in.
 A_s = reinforcement to resist flexure and/or direct tension, sq in.
 A_{sh} = hanger shear reinforcement across inclined dap crack, sq in.
 A_{s1} = corbel type reinforcement, beam ledge bottom, at beam end support, sq in.
 A_{s2} = beam web reinforcement, non-ledge side, at beam end support, sq in.
 A_t = beam torsion reinforcement, closed stirrups, ledge side of web, sq in.
 A_v = web shear stirrup reinforcement, sq in.
 A_{vh} = corbel type reinforcement, beam ledge, at beam end support, sq in.
 A_w = beam end 45 deg torsional equilibrium reinforcement ledge side of web, sq in.
 A_{wl} = beam end longitudinal component of A_w torsional equilibrium reinforcement ledge side of web, sq in.
 A_{wv} = beam end vertical component of A_w torsional equilibrium reinforcement ledge side of web, sq in.
 b_t = bearing width of concentrated ledge load, in.
 b_w = width of beam web, in.
 b_1 = effective width of ledge corbel at beam end support, in.
 c = clear concrete cover to reinforcement, in.
 d = distance from extreme compression fiber to centroid of flexural tension reinforcement, in.
 d_e = distance from end of beam to end concentrated load closest to beam end, in.
 d'_s = distance from centroid of tension reinforcement to extreme tension fiber, in.
 d_w = distance from non-ledge side of beam web to centroid of 45 deg A_w web reinforcement positioned closest to ledge side of beam, in.
 d_1 = distance from centroid of A_{s1} to top of ledge for corbel end support, in.
 d_2 = distance from beam web face, ledge side, to centroid of corbel end A_{s2} vertical web reinforcement, in.
 e_d = distance from beam end at top of ledge along an inclined 45 deg line on the web to a 90 deg intersecting line passing through the H_u equilibrium connection centroid (Fig. 21b), in.
 e_t = distance from 1 in. (25.4 mm) below the top of the concrete topping to the top surface of the beam ledge, in.
 e_w = perpendicular distance from the H_u equilibrium connection centroid to the inclined 45 deg line along which e_d is measured (Fig. 21b), in.
 e_1 = distance from beam ledge load, uniform or concentrated, to the beam shear center, in.
 e_2 = distance from beam end reaction to the beam shear center, in.
 f'_c = compressive strength of beam concrete, psi
 f_t = beam concrete combined flexural and tension stress, psi
 f_w = beam web flexure tension stress, psi
 f_y = yield strength of reinforcement or steel, psi or ksi
 h = vertical height of beam ledge, in.

- h_h = vertical distance from 1 in. (25.4 mm) below concrete topping to force centroid of beam H_u bottom equilibrium connection at beam end (Fig. 24b), in.
- h_w = vertical height of beam lower portion acting as a horizontal beam (Figs. 24b and 24d), in.
- H = horizontal applied load or horizontal torsion equilibrium connection force, working or ultimate, lbs or kips
- H_u = ultimate beam end horizontal equilibrium connection force, lbs or kips
- I_{max} = maximum principle moment of inertia, in.⁴
- I_{min} = minimum principle moment of inertia, in.⁴
- I_x = moment of inertia about x axis, in.⁴
- I_{xy} = product of inertia, in.⁴
- I_y = moment of inertia about y axis, in.⁴
- j_u = ratio of ultimate internal moment resisting arm to d , in.
- l_p = horizontal projection of beam ledge from web, in.
- L = span of beam, in.
- m = variable defining length of beam ledge attachment to web depending upon type and location of ledge load
- N = volume change horizontal load applied to beam, lbs or kips
- N_u = ultimate volume change horizontal load applied to beam, lbs or kips
- P = concentrated load applied to beam ledge, lbs or kips
- P_u = ultimate concentrated load applied to beam ledge, lbs or kips
- R, R_A, R_B = beam end support reaction, lbs or kips
- s = spacing between concentrated loads, in.
- S_{bp} = principle section modulus at beam bottom, in.³
- T, T_A, T_B = beam end equilibrium torque, in.-lbs or ft-kips
- V_u = ultimate shear load applied to beam ledge, lbs or kips
- w_{tu} = ultimate lateral loads per unit length applied to beam for beam torsional equilibrium (Fig. 24), lbs per ft
- Y_{bp} = distance from beam cross-sectional centroid to extreme bottom fiber along inclined vertical axis, in.
- λ = coefficient based upon unit weight of concrete, 1.0 for normal weight and 0.85 for sand-light-weight
- ϕ = ultimate strength reduction factor, 0.65 for plain concrete tension, 0.85 for shear, and 0.90 for reinforced flexure
- Σ = summation symbol
- θ = angle of inclination of principle inertia axes

APPENDIX—DESIGN EXAMPLES

The following three numerical examples address the design of a typical spandrel beam for a parking garage including the reinforcement requirements

for an end support corbel plus a sample calculation for determining the principle moments of inertia for a building spandrel beam.

EXAMPLE 1 — PARKING GARAGE SPANDREL BEAM

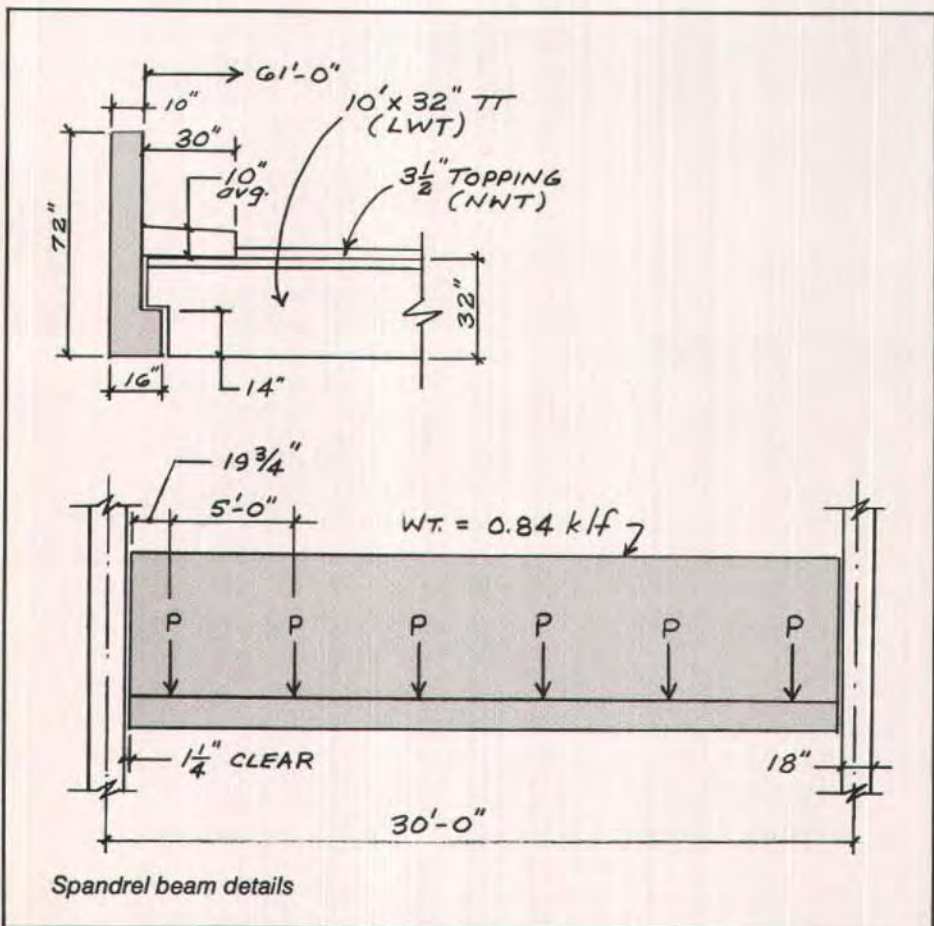
Spandrel Beam Data

Normal weight concrete
 $f'_c = 5000$ psi

Beam reinforcement

$f_y = 60$ ksi

The spandrel beam details are shown in the sketch.



Design Loads

$$\begin{aligned} \text{Live Load} &= \text{Weight of Cars} + \frac{1}{2} \text{ Snow Weight} \\ &= 50 + \frac{1}{2} (30) = 65 \text{ psf} \end{aligned}$$

Note that there is no live load reduction.

$$\begin{aligned} \text{Concentrated load per tee stem} \\ (D + L) &= 25.3 \text{ kips (working)} \end{aligned}$$

Find Shear Center

Refer to Fig. 20.

$$W_1 = 10 \text{ in.}, W_2 = 14 \text{ in.}$$

$$h_1 = 65 \text{ in.}, h_2 = 11 \text{ in.}$$

$$\begin{aligned} e_x &= \frac{11}{2} \left[\frac{11 (14)^3}{10 (65)^3 + 11 (14)^3} \right] \\ &= 0.060 \text{ in.} \end{aligned}$$

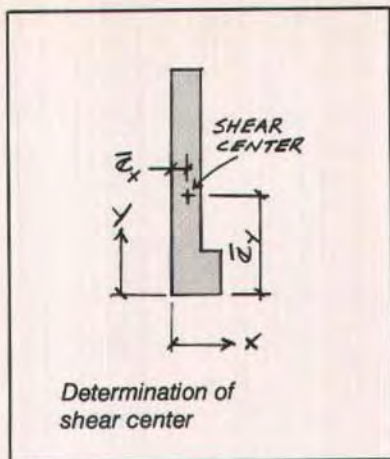
$$\bar{e}_x = \frac{10}{2} + 0.060 = 5.06 \text{ in.}$$

$$\begin{aligned} e_y &= \frac{65}{2} \left[\frac{65 (10)^3}{65 (10)^3 + 14 (11)^3} \right] \\ &= 25.26 \text{ in.} \end{aligned}$$

$$\bar{e}_y = \frac{14}{2} + 25.26 = 32.26 \text{ in.}$$

Location of beam center of gravity:

$$\bar{x} = 5.84 \text{ in.}, \bar{y} = 32.97 \text{ in.}$$



Beam End Design

Refer to Fig. 27b for beam-to-column details.

Design data:

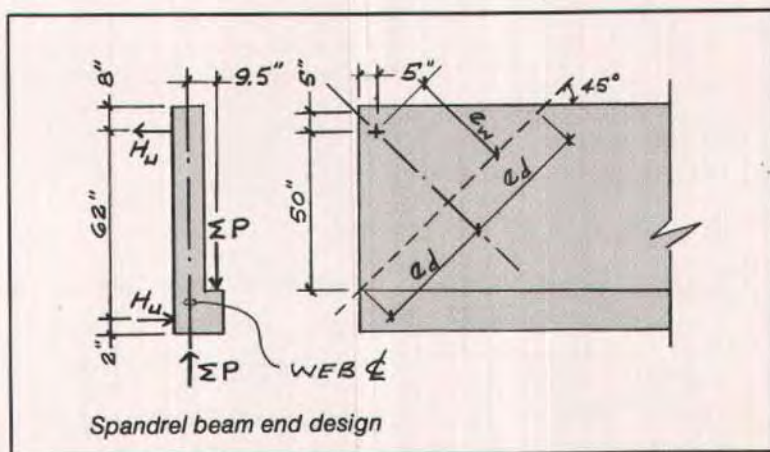
1. Refer to Fig. 21.
2. Consider shear center aligns with web centerline.
3. $\Sigma P = 3 (25.3) = 75.9 \text{ kips (working)}$

Find ultimate load factor:

$$P_{D+L} = 25.3 \text{ kips}$$

$$P_D = 15.4 \text{ kips}$$

$$P_L = 9.9 \text{ kips}$$



$$LF = \frac{1.4D + 1.7L}{D + L}$$

$$= \frac{1.4(15.4) + 1.7(9.9)}{25.3}$$

$$= 1.52$$

Find H_u web:

H_u forces to beam web, use $LF = 1.52$.

$$H_u = \frac{1.52(75.8)9.5}{62}$$

$$= 17.7 \text{ kips}$$

to top tension insert at ultimate.

Find e_d and e_w :

$$e_d = \frac{\sqrt{2}}{2}(50 + 5) = 38.89 \text{ in.}$$

$$2e_d = 2(38.89) = 77.8 \text{ in.}$$

$$e_w = \left(\frac{50 + 5}{2} - 5 \right) \sqrt{2} = 31.8 \text{ in.}$$

Determine A_w reinforcement:

Use Eq. (1). $d_w \approx 10 - 1.5 = 8.5 \text{ in.}$

$$A_w = \frac{17.7(31.8)}{0.85(8.5)60}$$

$$= 1.30 \text{ sq in. (minimum)}$$

Find minimum A_{wv} and A_{wt} :

Use Eqs. (2) and (3).

$$A_{wv} = A_{wt} = \frac{\sqrt{2}}{2} A_w = \frac{\sqrt{2}}{2} (1.30)$$

$$= 0.92 \text{ sq in.}$$

Distribute 0.92 sq in. horizontally over

$$\frac{\sqrt{2}}{2} (2e_d) = 55 \text{ in.}$$

Select #4 stirrups for A_{wv} and A_v .

$$A_v \text{ from: End to 1.65 ft} = 0.32 \text{ sq in./ft}$$

$$1.65 \text{ to 6.65 ft} = 0.17 \text{ sq in./ft}$$

Note that the above steel area includes the v_c reduction due to torsion.

$$A_{wv} + A_v = 0.92 + 0.32/2$$

$$= 1.08 \text{ sq in. (end to 1.65 ft)}$$

$$A_{wv} + A_v = 0.92 + 0.17/2$$

$$= 1.00 \text{ sq in. (1.65 to 6.65 ft)}$$

#4 stirrup spacing:

$$\frac{0.20}{1.08/55} = 10.2 \text{ in.}$$

A_{wt} bar size per 12 in. spacing:

$$12 \left(\frac{0.92}{55} \right) = 0.20 \text{ sq in.}$$

Use #4 bars at 12 in. center to center.

Determine tension insert ultimate loads:

1. Steel ultimate = $(4/3)(LF)$ (Working Load)
2. Concrete punching shear beam web and insert shear cone requires a load factor equal to three.
3. H_u Insert Steel:

$$(4/3)(26.8) = 35.7 \text{ kips}$$

4. H_u Concrete:

$$3 \left(\frac{26.8}{1.52} \right) = 52.9 \text{ kips}$$

Summary

1. Use #4 closed stirrups at 10 in. center-to-center over a length of 5 ft from beam end.
2. Use #4 bars for A_t at 12 in. center-to-center about beam perimeter. Use hairpin bars according to Fig. 25 at beam end.
3. Note that other design cases may control reinforcement.

Ledge Attachment Design

Determine ultimate stem load:

$$V_u = (4/3)(1.52)(25.3) = 51.3 \text{ kips}$$

Check end shear V_u for f_t at ultimate:

Use Fig. 22 and Eq. (4). $\frac{3}{4} l_p = 4.5 \text{ in.}$

$$d_{e_{max}} = 2h = 2(14) = 28 \text{ in.}$$

$$d_e = 19.75 \text{ in. controls } d_e \text{ selection}$$

$$s_{max} = b_t + 4h = 4.75 + 4(28) = 116.75 \text{ in.}$$

$$s = 60 \text{ in. controls}$$

$$m = d_e + s/2 = 19.75 + 60/2 = 49.75 \text{ in.}$$

$$f_t = \frac{51,300}{10(49.75)} + \frac{6(51,300)(10/2 + 4.5)}{(10)^2 49.75} \approx 691 \text{ psi}$$

f_t greater than

$$3\lambda\sqrt{f'_c} = 3(1)\sqrt{5000} = 212 \text{ psi}$$

Therefore, ledge reinforcement is required.

Select ledge attachment reinforcement at end V_u .

Use Eq. (5):

$$A_s = \frac{51.3(10 + 4.5 - 0.5)}{0.85(60)(10 - 1.5 - 0.5)} = 1.76 \text{ sq in. over length } m = 49.75 \text{ in.}$$

#4 bar spacing:

$$\frac{49.75}{1.76/0.20} = 5.65 \text{ in.}$$

Use #4 stirrups at 6 in. center-to-center over a length 4 ft 6 in. from beam end.

Determine inner v_u ledge attachment reinforcement:

$$V_u = 51.3 \text{ kips}$$

By inspection, if $f_t > 3\lambda\sqrt{f'_c}$, reinforcement is required.

Use Eq. (5) and previous example.

$$A_s = 1.76 \text{ sq in. over length } m = 60 \text{ in.}$$

#4 bars with a spacing of:

$$\frac{60}{1.76/0.20} = 6.82 \text{ in.}$$

Use #4 stirrups at 7 in. center-to-center starting at 4 ft 6 in. from each end.

Summary

1. Consideration should be given to increasing b_w to 12 in. if greater reinforcement spacing is required.
2. Note that other design cases may control.

Ledge Load Transfer Design

$V_u = 51.3$ kips per tee stem (see Fig. 23)

Concrete shear capacity inner V_u

$s = 60$ in. greater than

$$b_t + 2h = 4.75 + 2(14) = 32.75 \text{ in.}$$

Use Eq. (6)

$$V_u = \frac{3(0.85)}{1000} 14(1)\sqrt{5000} \times [2(6) + 4.75 + 14] = 77.6 \text{ kips}$$

Plain concrete shear capacity is greater than applied $V_u = 51.3$ kips. Therefore, no reinforcement per Eq. (11) is required for inner loads.

Concrete shear capacity end V_u :

$$d_e < 2h, d_e = 19.75 \text{ in., and } 2h = 28 \text{ in.}$$

Use Eq. (8):

$$V_u = \frac{0.85(14)}{1000} 1\sqrt{5000} \times [2(6) + (4.75 + 14)/2 + 19.75] = 34.6 \text{ kips}$$

Plain concrete shear capacity is less than applied $V_u = 51.3$ kips. Therefore, reinforcement per Eq. (11) is required for end V_u .

Select end V_u reinforcement:

Check applied shear stress.

$$\begin{aligned} \text{Shear Area} &= h(b_t + h) \\ &= 14(4.75 + 14) \\ &= 262.5 \text{ sq in.} \end{aligned}$$

$$v_u = \frac{51,300}{262.5} = 195 \text{ psi}$$

This stress is satisfactory since it is less than $10 \lambda \sqrt{f'_c} = 707 \text{ psi}$.

Use Eq. (11):

$$A_{sh} = \frac{51.3}{0.85(60)} = 1.01 \text{ sq in.}$$

over length $b_t + h = 18.75 \text{ in.}$

since $d_e > (b + h)/2$.

A_{sh} steel area provided by ledge reinforcement at 6 in. center-to-center for end V_u (see previous calculations) is:

$$0.20(18.75/6) = 0.62 \text{ sq in.}$$

Provide $1.01 - 0.62 = 0.39 \text{ sq in.}$ of additional steel reinforcement.

Use two #4 closed stirrups in addition to other reinforcement at each end V_u .

Determine $A_h = A_{sh}/2 = 1.01/2 = 0.50 \text{ sq in.}$

Select two #5 bars with end hairpins.

Select ledge flexural reinforcement.

Use Eq. (12):

$$d \approx h - 1.5 = 14 - 1.5 = 12.5 \text{ in.}$$

$$A_s = \frac{1}{0.85(60)} \left[51.3 \left(\frac{3}{4} \right) \frac{6}{12.5} + 0.15(51.3) \frac{14}{12.5} \right]$$

$$= 0.53 \text{ sq in. per tee stem load.}$$

Use #4 bars at 12 in. center-to-center with two additional bars at each tee stem distributed over $b_t + h = 18.75 \text{ in.}$

since $d_e > (b + h)/2$

Design Summary

See Fig. 27b for design details. Follow ACI 318-77 (Chapter 11) for shear and torsion reinforcement.

Reinforcement details:

Location from beam end	$A_v + 2A_t$	A_l
0 ft to 1 ft 7 $\frac{3}{4}$ in.	#4 bars at 5.6 in.	#4 bars at 12 in.*
1 ft 7 $\frac{3}{4}$ in. to 6 ft 7 $\frac{3}{4}$ in.	#4 bars at 10.4 in.	#3 bars at 12 in.*

Analysis of calculations:

A_v includes v_c reduction due to torsion.

Location from beam end	$A_v + 2A_t$	A_l
0 ft to 4 ft 1 $\frac{3}{4}$ in.	#4 bars at 5.6 in.	—
0 ft to 4 ft 7 in.	—	#4 bars at 12 in.*
4 ft 1 $\frac{3}{4}$ in. to midspan	#4 bars at 6.8 in.	

*At each web face.

Additional reinforcement:

1. Provide two additional #4 stirrups at end V_u tee stem loads.

2. Furnish #4 bars at 12 in. center-to-center plus two additional bars at each tee stem load for ledge flexural reinforcement.

EXAMPLE 2—PARKING GARAGE SPANDREL BEAM LEDGE CORBEL

Refer to Fig. 27c for spandrel beam details.

Design Data

1. See Example 1 for loads and dimensions.
2. The beam-to-column details are shown in Fig. 27c. The spandrel beam is torsionally stable without any end connections.
3. Design corbel end using method based on corbel behavior.
4. Refer to Fig. 25 for corbel requirements.
5. Consider ledge corbel loads to result only from applied tee stem loads.

Determine Ultimate Applied Loads

Ultimate shear load:

$$V_u = (4/3)(1.52)(3)(25.3) = 153.8 \text{ kips}$$

Ultimate shear stress:

Estimate $d_1 = 12.5$ in.

Corbel width $= 1.5h = 1.5(14) = 21$ in.

$$v_u = \frac{V_u}{b_1 d_1} = \frac{153,800}{21(12.5)} = 586 \text{ psi}$$

The calculated shear stress is therefore satisfactory since it is less than the recommended 800 psi maximum.

Find Ledge Corbel Reinforcement

Using Eq. (17), determine A_{s1} steel:

$$A_{s1} = \frac{3(153.8)6}{0.85(4)60(12.5)} = 1.09 \text{ sq in.}$$

This steel area controls. Select four #5 bars with welded cross-bars.

Using Eq. (18):

$$A_{s1} = \frac{(153.8)^2}{0.85(60)^2 2(1)21(12.5)} = 0.88 \text{ sq in.}$$

From Eq. (19):

$$A_{s1(\text{minimum})} = \frac{0.08(21)12.5}{60} = 0.35 \text{ sq in.}$$

Using Eq. (20), determine A_{vh} steel:

$$A_{vh} = \frac{153.8}{0.85(60)4(1)21(12.5)} = 0.44 \text{ sq in. (controls)}$$

From Eq. (21):

$$A_{vh(\text{minimum})} = \frac{0.04(21)12.5}{60} = 0.18 \text{ sq in.}$$

Distribute A_{vh} per Fig. 25c and consider only one leg (lower) of closed stirrup effective (see Fig. 25d).

Spacing for #4 bars:

$$\frac{1.5 b_1 (\text{bar area})}{A_{vh}} = \frac{21(0.20)}{0.44} = 9.6 \text{ in.}$$

Maximum spacing:

$$h/2 = 14/2 = 7 \text{ in. center-to-center}$$

Using Eq. (22) determine A_{s2} steel:

Estimate $d_2 = 10 - 1.5 = 8.5$ in.

$$A_{s2} = \frac{12.5}{8.5}(1.09) = 1.60 \text{ sq in.}$$

Select #4 bars.

Total required $= (1.60/0.20) = 8$ bars

Over $1.5h = 21$ in.

Note that no ledge attachment reinforcement is required due to beam reaction for end tee stem V_u .

Add #4 stirrups so that additional stirrups plus shear and torsion stirrups provide 1.60 sq in. over the beam's end 21 in.

Design Summary

1. Cross-bar welds (#5 cross-bar) for the four #5 A_{v1} bars should be in accordance with the PCI Design Handbook requirements and completely specified.
2. Analysis of bearing pad bearing

stresses at a working load of 3 (25.3) = 75.9 kips is required considering the bearing is not uniform.

3. The ultimate concrete bearing stress on the extreme edge of the ledge may require the use of additional bearing reinforcement according to Fig. 39.

EXAMPLE 3 — DETERMINATION OF PRINCIPLE MOMENTS OF INERTIA FOR A BUILDING SPANDREL BEAM

Assume a spandrel beam for a building with a configuration and dimensions shown in cross section below. Divide beam into two parts.

Beam Properties

$$\begin{aligned}\bar{x} &= 8.12 \text{ in.} \\ \bar{y} &= 15.66 \text{ in.} \\ I_x &= 54,024 \text{ in.}^4 \\ S_b &= 3450 \text{ in.}^3 \\ I_y &= 13,283 \text{ in.}^4 \\ A_1 &= 476 \text{ sq in.} \\ A_2 &= 60 \text{ sq in.}\end{aligned}$$

Determine Product of Inertia I_{xy}

From strength of materials:

$$\begin{aligned}I_{xy} &= \sum x_i y_i A_i \\ I_{xy} &= x_1 y_1 A_1 + x_2 y_2 A_2 \\ x_1 &= 7 - 8.12 = -1.12 \text{ in.} \\ y_1 &= 17 - 15.66 = +1.34 \text{ in.} \\ x_2 &= 17 - 8.12 = +8.88 \text{ in.} \\ y_2 &= 5 - 15.66 = -10.66 \text{ in.} \\ I_{xy} &= (-1.12)(+1.34)476 + (+8.88) \times \\ &\quad (-10.66)60 = -6394 \text{ in.}^4\end{aligned}$$

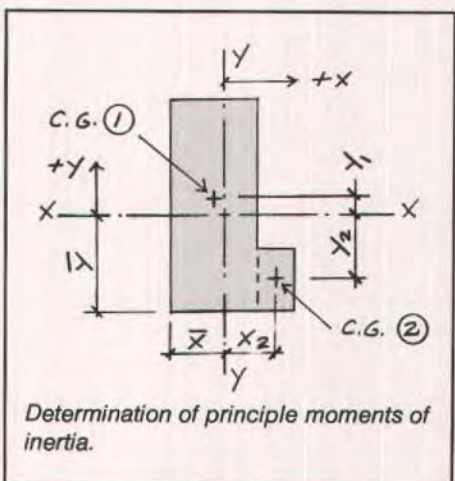
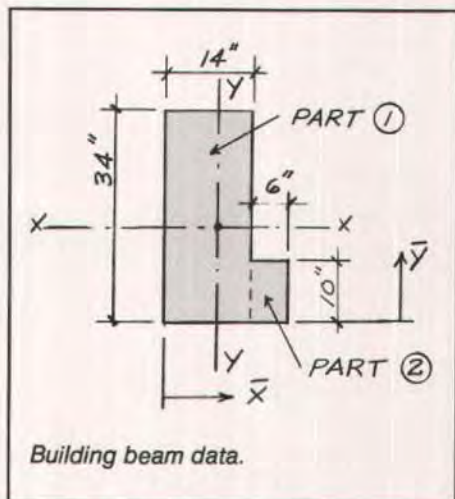
Find Orientation Angle θ

Refer to Fig. 26 and use Eq. (23):

$$\begin{aligned}\theta &= \frac{1}{2} \tan^{-1} \left[\frac{2(-6394)}{13,283 - 54,024} \right] \\ &= +8.71 \text{ deg (counterclockwise)}\end{aligned}$$

Find I_{max}

Use Eq. (25):



$$I_{max} = \frac{54,024 + 13,283}{2} + \sqrt{\left(\frac{54,024 - 13,283}{2}\right)^2 + (-6394)^2}$$

$$= 55,003 \text{ in.}^4$$

Find I_{min}

Use Eq. (26):

$$I_{min} = \frac{54,024 + 13,283}{2} - \sqrt{\left(\frac{54,024 - 13,283}{2}\right)^2 + (-6394)^2}$$

$$= 12,304 \text{ in.}^4$$

Find $S_{b \min}$ Associated With I_{max}

$$a = \sqrt{(15.66)^2 + (11.88)^2}$$

$$= 19.66 \text{ in.}$$

$$\beta = \sin^{-1}\left(\frac{11.88}{19.66}\right)$$

$$= 37.18 \text{ deg}$$

$$\beta - \theta = 37.18 - 8.71$$

$$= 28.47 \text{ deg}$$

$$y_{bp} = (a) \cos(\beta - \theta)$$

$$= 19.66 \cos(28.47)$$

$$= 17.28 \text{ in.}$$

Determine $S_{b \min}$ at Point A

$$S_{b \min} = \frac{I_{max}}{y_{bp}}$$

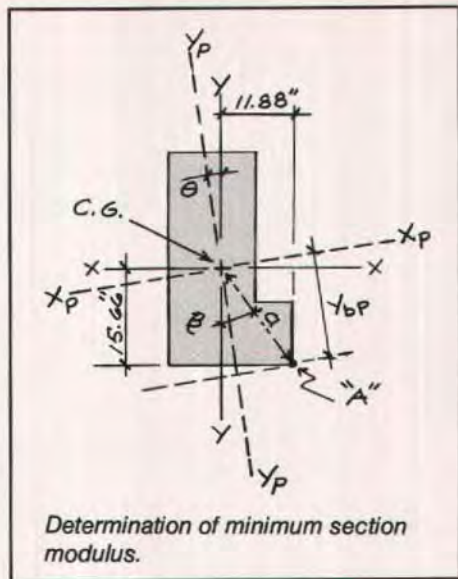
$$= \frac{55,003}{17.28}$$

$$= 3188 \text{ in.}^3$$

which is less than the value of $S_b = 3450 \text{ in.}^3$ for I_x found previously.

Summary

1. If the beam is prestressed, the internal prestress moment is the product of the strand force times the distance



from the beam center of gravity to the strand center of gravity where the distance is parallel to line $y_p - y_p$.

2. The above procedure for unsymmetrical members can be used generally for all precast prestressed concrete members.
3. A graphical approach is recommended for determining all dimensions such as y_{bp} once θ , I_{max} , and I_{min} have been calculated.

METRIC (SI) CONVERSION FACTORS

- 1 ft = 0.305 m
- 1 sq ft = 0.0929 m²
- 1 in. = 25.4 mm
- 1 sq in. = 645.2 mm²
- 1 in.³ = 16,388 mm³
- 1 in.⁴ = 42,077 mm⁴
- 1 kip = 4.448 kN
- 1 lb = 4.448 N
- 1 ksi = 6.895 MPa
- 1 psi = 0.006895 MPa
- 1 psf = 0.04788 kPa

* * *